The Universal Clock Carrier

Boundary-Selected CMC Ordering in the Two-Clock STF Framework

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Every claim in this paper carries a status per the framework's taxonomy (First Principles V8.1 §I.E): ***theorem*** *·* ***derived*** *·* ***conditional*** *·* ***numerical*** *·* ***commitment*** *·* ***open****. The ledger of §21 uses these terms; its "false" entries are refuted routes recorded with their reasons.*

Abstract

The Clock-Separation Theorem proves that an oscillating scalar amplitude cannot also generate a continuous global time direction through its normalized gradient. The Selective Transient Field (STF) therefore requires two distinct temporal objects: an internal cyclic phase $\Theta_I \in S^1$ carried by the oscillating field, and a universal ordering with future unit normal

$$N^\mu = -\frac{\nabla^\mu T_U}{\sqrt{-\nabla T_U \cdot \nabla T_U}}.$$

This paper investigates what can carry the universal ordering without adding an unjustified propagating khronon. A sequence of obstruction and construction results selects a conditional answer. The first closure must be a spacelike boundary rather than an event. Lorentzian distance from that boundary gives a canonical local proper-time clock, but caustics obstruct its generic global extension and its unit-eikonal form forces a geodesic lapse. A curvature gradient from the compactified parent supplies a covariant first-closure seed on evolving FLRW, including radiation when the Kretschmann parent is used, but fails in de Sitter and produces unacceptable higher-derivative bulk terms when substituted directly into the local STF action.

The strongest surviving release mechanism is geometric. Extremizing the three-volume of each leaf at fixed enclosed four-volume gives constant mean curvature (CMC). The associated lapse equation is elliptic, not a clock wave; the foliation permits spatial lapse gradients and continues after the curvature seed becomes stationary. Accumulated four-volume orders the leaves without identifying their numerical label with an observable clock rate.

Two further results delimit this construction. First, a generic pointwise curvature-threshold surface is not CMC. For the ordinary matter growing mode, the Seed–Release Junction Functional is strictly positive for every nonzero wavenumber. Second, the local surface-volume action is the cuscuton/CMC action, but promoting the STF rate interaction to a

dynamical normalized-gradient clock generically forces the leaves away from CMC. The clock dependence of the Bel–Robinson norm begins at second perturbative order on $R_0 \neq 0$ FLRW and cannot cancel the linear obstruction.

The resulting candidate is not a new local scalar field. It is a boundary-selected, metric-defined functional

$$N^\mu = N^\mu_{\mathrm{CMC}}[g; \Sigma_{\mathrm{closure}}, \Sigma_{\mathrm{terminal}}, V_4],$$

with an explicit finite seed-to-release transition where required. It is diffeomorphism covariant but spatially nonlocal, introduces no independently specifiable clock wave, and leaves the internal STF phase untouched. Global CMC existence, uniqueness, the nonlocal metric variation, and the causal memory completion remain open. Universal time is thus carried most economically by global closure geometry, while local closed systems carry the cyclic clocks by which that universal ordering is read.

1. The problem fixed by the two-clock theorem

Write the oscillating STF field as

$$\phi = A\cos\Theta_I, \qquad \Theta_I \simeq m_s T_U + \delta \pmod{2\pi}.$$

The internal state recurs after one winding. Its amplitude reaches a turning point whenever

$$\frac{d\phi}{dT_U} = -A m_s \sin\Theta_I = 0,$$

while

$$\frac{d\Theta_I}{dT_U} = m_s$$

remains nonzero. The amplitude is therefore only a local chart of the cyclic phase.

More generally, let q have a nonvanishing timelike gradient and define

$$n_q^\mu = s\frac{\nabla^\mu q}{\sqrt{-\nabla q\cdot\nabla q}}, \qquad s = \pm 1.$$

Then

$$n_q^\mu \nabla_\mu q = -s\sqrt{-\nabla q\cdot\nabla q}$$

has fixed nonzero sign. The generating scalar is strictly monotone along every integral curve. A periodic scalar cannot be a global normalized-gradient clock.

This theorem does not establish two independent propagating fields. It establishes two distinct temporal roles:

$$\boxed{\begin{aligned}\Theta_I \in S^1 &: \text{internal recurrence and local phase,}\\ T_U \in \mathbb{R} &: \text{universal ordering and future orientation.}\end{aligned}}$$

The carrier problem is to construct the second role without reassigning it to the first.

2. Requirements on a universal carrier

A viable STF carrier must satisfy all of the following.

1. It must define a future timelike, nonvanishing, hypersurface-orthogonal normal N^μ throughout the response domain.
2. It must remain defined while ϕ and any other oscillating amplitude pass through turning points.
3. It must reduce to the comoving normal on exact FLRW.

4. It must survive a late-time limit in which curvature scalars become stationary.
5. It must permit spatial clock-rate gradients when the geometry requires them; a forced unit-proper-time congruence is too restrictive.
6. It should not add a freely propagating universal-clock wave without a UV derivation and stability analysis.
7. It must enter the metric variation consistently. A preferred foliation may be covariant, but it is physical and contributes to the constraint algebra.
8. It must distinguish the ordering of leaves from any conventional numerical label assigned to them.
9. It must specify global branch selection and not merely prove the existence of some time function.
10. It must be compatible with the causal, zero-mode-subtracted STF response, not only with its local low-frequency approximation.

These requirements rule out several superficially economical carriers.

3. Carrier routes closed or restricted

3.1 The oscillating amplitude

Closed by the Clock-Separation Theorem. The normalized gradient fails at every turning point and reverses chart orientation across it.

3.2 A one-modulus axionic phase

For a complex modulus

$$\mathcal{T} = \tau + i\vartheta, \qquad K = -3\ln(\mathcal{T} + \bar{\mathcal{T}}),$$

the compactification normalization consistent with $\mathcal{V}_6 \propto e^{6\sigma} \propto \tau^{3/2}$ is

$$\tau = e^{4\sigma}, \qquad \phi = \sqrt{\frac{3}{2}} M_{\rm Pl} \ln \tau = \sqrt{24} M_{\rm Pl} \sigma.$$

The exponent and normalization independently confirm the published First Principles V8.1 §II.D correction (obtained there by canonical kinetic normalization, audit 22.2; here by internal-volume/Kähler consistency) — two routes, one answer. If the axion is exactly shift symmetric, its homogeneous charge obeys

$$Q = a^3 f_\vartheta^2 \dot{\vartheta} = \text{constant}.$$

A nonzero Q carries a local clock direction through radial turning points, but after radial stabilization

$$\dot{\vartheta} \propto a^{-3}, \qquad \rho_\vartheta \propto a^{-6}.$$

An unsourced shift charge cannot maintain both a nonvanishing asymptotic clock gradient and a nonvanishing canonical normalization through unlimited expansion. Conversely, a genuine one-modulus KKLT potential contains $e^{-a\mathcal{T}}$ and stabilizes the axion as well as the radial modulus.

The axionic route remains a possible finite-epoch local phase reference. It does not furnish the global universal carrier by itself.

Corpus consequence, stated explicitly. The dilution theorem and the KKLT periodicity close the *unwrapped STF phase* as a global carrier — the candidate the Clock-Separation paper called the most natively STF completion and First Principles V8.1 Appendix B.7 lists among five. With this paper, the five-candidate fork of the framework's theorem inventory resolves as: oscillating amplitude — closed by theorem (§3.1); unwrapped/axionic phase — closed as global carrier (§3.2), retained as a finite-epoch local reference; unit-eikonal/minimal multiplier — closed (§3.3); an independently propagating khronon — excluded by requirement 6 unless UV-derived; boundary-defined ordering — this paper's construction, conditional (§16). The carrier question is thereby reduced from a five-way fork to one conditional construction plus its named open items.

3.3 The unit-eikonal clock

The minimal multiplier action

$$S_{\text{unit}} = \int \sqrt{-g}\,\lambda_U(\nabla T_U \cdot \nabla T_U + 1)$$

enforces unit proper time. Its normal congruence is geodesic and its multiplier obeys a dust-like continuity equation. The resulting lapse has no spatial gradient:

$$a_i = D_i \ln \alpha = 0.$$

This is incompatible with using spatial clock-rate variation as a physical response channel. Normal geodesics also generically develop caustics. This reproduces the $q = 1$ obstruction recorded in First Principles V8.1 Appendix B.7 ($A_\mu^{(N)} = D_\mu^\perp \ln \alpha$; the unit-norm multiplier forces a geodesic universal flow) — agreement between independent derivations.

3.4 A curvature scalar as the bulk clock

A parent invariant can select an initial leaf, but direct substitution of

$$N_\mu \propto -\nabla_\mu \sqrt{\mathcal{I}_4}$$

into the bulk local rate interaction promotes second metric derivatives inside the clock normal. At quadratic order the directional response contains a generic term of the form

$$\kappa \int a\,\phi_0 C_w (\partial_i \ddot{\Psi})^2,$$

with nonzero C_w for generic constant equation of state. This exposes a candidate nondegenerate higher-derivative mode. The curvature scalar should seed the clock, not carry it through the entire bulk.

4. First closure as a spacelike boundary

Universal ordering cannot begin at one spacetime event. A single event has a future light cone, not a global simultaneous boundary. The first global activation must be represented by a spacelike achronal hypersurface

$$\Sigma_{\text{cl}}.$$

Given a smooth spacelike Cauchy surface, the local Lorentzian distance

$$T_\Sigma(p) = \sup_{q \in \Sigma_{\text{cl}}} L(q, p)$$

satisfies, before the cut locus,

$$\nabla_\mu T_\Sigma \nabla^\mu T_\Sigma = -1.$$

Its normalized gradient gives the future normal and crosses every turning point of ϕ. The construction is covariant and introduces no new wave field. **Status: theorem (before the cut locus).**

It is only local. Intersecting normal geodesics produce caustics and a cut locus where the distance function ceases to be smooth. Global hyperbolicity guarantees the existence of smooth temporal functions under standard hypotheses, but does not uniquely select one from the first boundary. Boundary selection and foliation selection are separate theorems.

5. The curvature seed

The compactified parent naturally supplies the quadratic invariant

$$\mathcal{I}_4 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}.$$

Define

$$Q := \sqrt{\mathcal{I}_4}, \qquad \Xi_Q := \sqrt{-\nabla Q \cdot \nabla Q}.$$

For spatially flat constant-w FLRW,

$$\mathcal{I}_4 = 3\left[(1+3w)^2 + 4\right] H^4$$

and

$$\Xi_Q = 3(1+w)\sqrt{3[(1+3w)^2+4]}\, H^3.$$

For an expanding branch with $w > -1$, the gradient is timelike and selects the comoving normal. Unlike the Ricci scalar, Q remains nonzero in exact radiation FLRW:

$$\mathcal{I}_4 = 24H^4 \qquad (w = 1/3).$$

The first-closure seed may therefore be defined covariantly by

$$\Sigma_{\rm cl} = \{\Xi_Q = \Xi_{\rm crit}\}.$$

Status: derived (the FLRW seed and its radiation-era survival; verified).

This construction has two limitations.

1. In exact de Sitter, $\dot{Q} = 0$, so it cannot continue to carry the clock.
2. A perturbed threshold leaf is not generically CMC, so it cannot immediately launch the preferred release foliation.

The parent curvature supplies orientation and an initial geometric surface. A different rule must release that surface into the bulk.

6. The CMC-volume selection principle

Let Σ be a spacelike hypersurface with induced metric h_{ij}, future unit normal N^μ, and mean curvature

$$K = \nabla_\mu N^\mu.$$

Under a normal deformation $\delta X^\mu = f N^\mu$, choose the orientation in which

$$\delta V_3 = \int_\Sigma d^3x \sqrt{h}\, K f, \qquad \delta V_4 = \int_\Sigma d^3x \sqrt{h}\, f.$$

Extremizing leaf volume at fixed enclosed four-volume gives

$$\delta(V_3 - \lambda V_4) = 0$$

and hence

$$\boxed{K = \lambda = \text{constant on each leaf.}}$$

This is the first direct variational bridge between STF closure/capacity language and a foliation equation. It motivates the following commitment:

> At a fixed accumulated amount of post-closure spacetime, universal closure selects an extremal spatial boundary.

The geometric implication fixed-volume extremum $\Rightarrow$ CMC is proved (**theorem**). The statement that STF closure enforces this extremum is the constitutive identification proposed here — the **CMC-Volume Closure Commitment** (**status: commitment**), priced so it can be refused: a reader who declines it retains everything below — the elliptic lapse, the junction analysis, the obstruction results — but is left with a well-defined foliation *family* rather than an STF-selected member, and the carrier reverts from a conditional answer to a consistent candidate class.

7. The CMC lapse is elliptic

In ADM form,

$$ds^2 = -\alpha^2 d\tau^2 + h_{ij}(dx^i + \beta^i d\tau)(dx^j + \beta^j d\tau),$$

mean curvature evolves as

$$(\partial_\tau - \mathcal{L}_\beta)K = D^2\alpha - \alpha(K_{ij}K^{ij} + R_{NN}).$$

On a CMC leaf, the prescribed spatially constant change in K gives

$$\boxed{D^2\alpha - \alpha(K_{ij}K^{ij} + R_{NN}) = F(\tau).}$$

This is an elliptic equation on each leaf (**status: standard geometric result**). It determines the lapse from the spatial geometry and boundary data rather than from a second time derivative. The normal acceleration

$$a_i = D_i \ln \alpha$$

is generically nonzero. The carrier therefore permits spatial clock-rate gradients without introducing independent clock-wave Cauchy data.

On exact flat FLRW,

$$K = 3H, \qquad K_{ij}K^{ij} + R_{NN} = -3\dot{H},$$

and $\alpha = 1$ solves the lapse equation. Thus

$$N_{\text{symmetry}} = N_{\nabla Q} = N_{\text{CMC}}$$

on the evolving homogeneous background.

8. Global ordering by accumulated four-volume

Let the post-closure domain be bounded by an initial closure surface Σ_- and a terminal macroboundary Σ_+. Given a selected CMC foliation $\{\Sigma_s\}$, define

$$v(s) = \frac{V_4(\Sigma_-, \Sigma_s)}{V_4(\Sigma_-, \Sigma_+)}.$$

Then $v = 0$ on the initial boundary and $v = 1$ on the terminal boundary. Any monotone relabelling

$$T_U = f(v), \qquad f' > 0,$$

selects the same leaves and normalized normal.

The geometry determines ordering; it does not determine an observable rate $dT_U/d\tau_O$ against a laboratory clock. This preserves the Clock-Rate Invisibility Lemma and keeps the measurement problem distinct from the carrier problem.

In exact de Sitter, all standard flat leaves have the same $K = 3H$, so mean curvature alone cannot order them. The inherited branch and accumulated four-volume continue to distinguish the leaves after the curvature seed has become stationary.

9. The seed–release junction

The threshold seed is displaced by scalar perturbations. Let

$$Q = Q_0(t) + \delta Q(t, \mathbf{x}).$$

In Newtonian gauge, the threshold-leaf displacement is

$$\boxed{\pi_\Xi = \frac{\delta\dot{Q} - \Phi\dot{Q}_0}{\ddot{Q}_0}.}$$

For a general displaced leaf $t + \pi = \text{constant}$, the pulled-back mean-curvature perturbation is

$$\boxed{\delta K_{\text{leaf}} = -3(\dot{\Psi} + H\Phi) - \frac{1}{a^2}\nabla^2\pi - \dot{K}_0\pi.}$$

For a nonzero Fourier mode, the curvature seed launches a CMC leaf only if

$$\boxed{\mathcal{J}_k := -3(\dot{\Psi}_k + H\Phi_k) + \left(\frac{k^2}{a^2} - \dot{K}_0\right)\frac{\delta\dot{Q}_k - \Phi_k\dot{Q}_0}{\ddot{Q}_0} = 0.}$$

The threshold condition fixes π_Ξ; it does not impose $\mathcal{J}_k = 0$. **Status: derived** (the junction functional below is exact at linear order and has been independently re-derived from a full perturbed-metric computation).

For the ordinary GR matter growing mode,

$$\Phi_k = \Psi_k = \text{constant}, \qquad y = \frac{k^2}{a^2H^2},$$

one finds

$$\boxed{\frac{\mathcal{J}_k}{H\Phi_k} = \frac{10}{9}y + \frac{8}{81}y^2 > 0 \qquad (k \neq 0).}$$

Ordinary Einstein constraints do not make the pointwise curvature seed CMC. The mismatch vanishes only in the homogeneous separate-universe limit.

10. The seed-to-release transition

The natural geometric projection is volume-preserving mean-curvature flow. Let

$$\bar{K} = \frac{1}{V_3}\int_\Sigma d^3x\sqrt{h}\,K$$

and deform the seed with normal speed

$$f = -(K - \bar{K}).$$

Then

$$\delta V_4 = 0$$

and

$$\boxed{\delta V_3 = -\int_\Sigma d^3x\sqrt{h}\,(K - \bar{K})^2 \leq 0.}$$

The flow preserves enclosed four-volume while monotonically relaxing the leaf toward CMC where a regular limit exists. **Status: derived** (the monotonicity identity); the existence of a regular limit for the STF solution class is **open**.

Its cost is physical: the resulting CMC representative is generally no longer the pointwise surface $\Xi_Q = \Xi_{\rm crit}$. STF must therefore distinguish

$$\boxed{\text{local activation seed} \quad \text{from} \quad \text{global universal-clock release.}}$$

The construction proposed here is

$$\boxed{\text{curvature seed} \longrightarrow \text{finite volume-preserving geometric transition} \longrightarrow \text{CMC-volume ordering.}}$$

The thickness and dynamics of the transition remain to be derived. Treating the flow parameter as physical time would introduce a third temporal object; it should instead be understood as a variational projection or a finite constitutive boundary layer.

11. Why the internal STF scalar cannot close the junction

The canonical scalar stress is unambiguous:

$$\delta\rho_\phi = \dot\phi_0\delta\dot\phi - \dot\phi_0^2\Phi + m_s^2\phi_0\delta\phi,$$

$$\delta p_\phi = \dot\phi_0\delta\dot\phi - \dot\phi_0^2\Phi - m_s^2\phi_0\delta\phi.$$

Its perturbation satisfies a driven second-order Klein–Gordon equation and retains two homogeneous initial data per mode. The CMC junction is an additional elliptic relation,

$$\pi_k^{\rm CMC} = \frac{3(\dot\Psi_k + H\Phi_k)}{k^2/a^2 - \dot K_0}.$$

A propagating scalar can participate in a specially selected solution that satisfies this condition. It cannot enforce it for every solution unless a new multiplier or degeneracy removes one combination of its physical data.

Making the internal oscillator perform the universal clock's constraint work would collapse the two-clock separation dynamically even if the symbols remained distinct.

12. The local surface-volume correspondence

A local action whose constant-field surfaces are CMC is

$$\boxed{S_U^{\rm local} = \int d^4x\sqrt{-g}\,[\sigma_U q_U - U(T_U)], \qquad q_U = \sqrt{-\nabla T_U^2}.}$$

Variation gives

$$\boxed{\sigma_U K + U'(T_U) = 0.}$$

This is the cuscuton/CMC structure. **Status: derived correspondence.** Through the coarea identity,

$$\int \sqrt{-g}\,q_U = \int dT_U \int_{\Sigma_{T_U}} \sqrt{h},$$

its square-root term is a continuous sum of leaf three-volumes, while the potential weights the intervening four-volume.

The correspondence is important but does not make the local cuscuton the STF carrier automatically.

1. The carrier stress is nonzero:

$$T_{\mu\nu}^{(U)} = (\sigma_U q_U - U)g_{\mu\nu} + \sigma_U q_U N_\mu N_\nu.$$

2. Fixed σ_U and $U(T_U)$ select a normalization of the clock label rather than preserving arbitrary relabelling as a gauge symmetry.
3. Generic Hamiltonian realizations require a fresh degree-of-freedom audit; the constraint-like CMC branch cannot be assumed from the class name.
4. Most decisively, the STF interaction backreacts on the local clock equation.

13. Full clock variation of the local STF interaction

Let

$$S(N) = \mathcal{R}_{\rm STF}(N) = \sqrt{R^2 + 8\mathcal{W}_N}, \qquad \mathcal{W}_N = E_{\mu\nu}E^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}.$$

Define

$$\Pi_\alpha := \frac{\partial S}{\partial N^\alpha}\bigg|_{g,C}.$$

For the local rate interaction

$$S_{\rm int} = \int \sqrt{-g}\,\kappa\phi N^\mu \nabla_\mu S(N),$$

the exact variation with respect to T_U is

$$\boxed{E^{\rm int}_{T_U} = \nabla_\nu\left\{\frac{h^{\nu\alpha}}{q_U}[\kappa\phi\nabla_\alpha S - \Pi_\alpha \nabla_\mu(\kappa\phi N^\mu)]\right\}.}$$

On exact FLRW the Weyl tensor vanishes. For $R_0 \neq 0$,

$$E_{\mu\nu}, B_{\mu\nu} = O(\epsilon), \qquad \mathcal{W}_N = O(\epsilon^2), \qquad \Pi_\alpha = O(\epsilon^2).$$

The explicit Bel–Robinson clock dependence cannot cancel a linear clock source.

For a displaced clock foliation $t + \pi_U = \text{constant}$, the source is

$$\boxed{\delta E^{\rm int}_{T_U,k} = -\frac{\kappa\phi_0}{q_{U0}}\frac{k^2}{a^2}(\delta S_k - \dot{S}_0 \pi_{U,k}).}$$

The bracket is the curvature perturbation pulled back to the physical leaf. The coupled local carrier obeys

$$\boxed{\sigma_U[-3A_k + (x - \dot{K}_0)\pi_{U,k}] = -C_U x(\delta S_k - \dot{S}_0\pi_{U,k}),}$$

where

$$A_k = \dot{\Psi}_k + H\Phi_k, \qquad x = k^2/a^2, \qquad C_U = \kappa\phi_0/q_{U0}.$$

It follows that

$$\pi_{U,k} = \frac{3\sigma_U A_k - C_U x\delta S_k}{\sigma_U(x - \dot{K}_0) - C_U x\dot{S}_0}.$$

This is a curvature-forced mean-curvature foliation, not CMC. **Status: derived.**

14. The local-carrier compatibility obstruction

If exact CMC is imposed on the coupled local carrier, then for every active nonzero mode

$$\delta S_k - \dot{S}_0 \pi_{U,k} = 0.$$

The leaf must be simultaneously CMC and uniform in STF curvature:

$$D_i K = 0, \qquad D_i S = 0.$$

For the matter growing mode,

$$\pi_{S,k} = \frac{\Phi_k}{H}\left(\frac{2}{3} + \frac{2}{9}y\right),$$

whereas

$$\pi_{\mathrm{CMC,k}} = \frac{\Phi_k}{H}\frac{3}{y+9/2}.$$

Equality gives

$$\boxed{y(2y+15) = 0.}$$

The only nonnegative solution is $y = 0$. The minimal local cuscuton carrier plus the unmodified local STF rate coupling therefore fails exact-CMC compatibility on generic inhomogeneous FLRW perturbations. **Status: derived** (independently re-derived; the displayed $\pi_{S,k}$ and $\pi_{\mathrm{CMC},k}$ reproduce exactly and the incompatibility follows from them algebraically). Per the framework's frontier discipline this obstruction is recorded, not chased: it closes the minimal local branch and is consistent with the published regime limitation of the completed-action analysis.

Higher-order Bel–Robinson terms cannot cancel a nonzero linear coefficient. This closes that minimal local branch for exact CMC.

15. The radiation differentiability boundary

In exact radiation FLRW,

$$R_0 = 0, \qquad C^{(0)} = 0, \qquad S_0 = 0.$$

The first perturbed magnitude is

$$S = \sqrt{(\delta R)^2 + 8\mathcal{W}_N^{(2)}}.$$

This expression is first-order homogeneous but not a linear functional. It has no Fréchet derivative at the origin. A standard quadratic perturbation action therefore requires one of the following:

1. a derived regularization of the norm;
2. a directional branch prescription;
3. use of the unsquared quadratic invariant through the radiation transition;
4. a nonsmooth constitutive theory formulated without ordinary linearization.

Exact radiation may remain invisible to the response channel, but perturbation theory about that origin is not automatically defined.

16. The surviving carrier: a metric-defined boundary functional

The preceding results favor a carrier that is geometric and nonlocal rather than an independently varied local scalar:

$$\boxed{N^\mu = N^\mu_{\mathrm{CMC}}[g; \Sigma_-, \Sigma_+, V_4].}$$

The construction is:

1. the parent-curvature threshold supplies an initial seed and time orientation;
2. a finite volume-preserving projection maps a generic seed to its CMC representative;
3. the initial and terminal boundaries select the global CMC branch;
4. the elliptic CMC equation determines the lapse and normal on each leaf;
5. accumulated four-volume orders the leaves;
6. the fixed-clock STF response is evaluated on this selected normal;
7. the internal phase remains a separate propagating cyclic degree of freedom.

This route is covariant if the metric and boundary data transform together. It selects a physical preferred foliation without inserting a nondynamical coordinate frame. **Status: conditional** — on the CMC-Volume Closure Commitment (§6) and on the open items of §19 and §23.

It is also explicitly nonlocal. The functional derivative of the response action contains

$$\boxed{\frac{\delta S_{\rm STF}}{\delta g_{\mu\nu}} = \left(\frac{\partial S_{\rm STF}}{\partial g_{\mu\nu}}\right)_N + \frac{\delta S_{\rm STF}}{\delta N^\alpha}\frac{\delta N^\alpha_{\rm CMC}}{\delta g_{\mu\nu}}.}$$

The second term is essential. Omitting it would return to the incomplete fixed-clock metric variation and generally violate the diffeomorphism Ward identity.

17. First linear response of the metric-defined clock

At scalar order the CMC condition itself gives the metric response. For every nonzero mode,

$$\boxed{\pi_{{\rm CMC},k} = \frac{3(\dot\Psi_k + H\Phi_k)}{k^2/a^2 - \dot K_0}.}$$

Since

$$N_{i,k} = -ik_i\pi_{{\rm CMC},k},$$

one obtains

$$\boxed{N_{i,k} = -ik_i\frac{3(\dot\Psi_k + H\Phi_k)}{k^2/a^2 - \dot K_0}.}$$

This is the leading functional response $\delta N_{\rm CMC}/\delta g$ (**status: derived**, scalar FLRW order). Its inverse Helmholtz-type denominator is the Fourier representation of the elliptic CMC operator. It is spatially nonlocal and temporally constraint-like.

The fully covariant response follows from the implicit-function relation

$$\mathcal{F}_{\rm CMC}[g, T_U] = D_i K = 0.$$

Formally,

$$\boxed{\delta T_U = -\left(\frac{\delta\mathcal{F}_{\rm CMC}}{\delta T_U}\right)^{-1}\frac{\delta\mathcal{F}_{\rm CMC}}{\delta g_{\mu\nu}}\delta g_{\mu\nu}.}$$

The inverse is the CMC lapse/leaf operator with the chosen boundary conditions. This equation identifies precisely what the completed metric variation must calculate.

18. Covariance, causality, and degrees of freedom

Covariance

The construction uses only geometric boundaries, induced volumes, mean curvature, and the metric. It is diffeomorphism covariant even though it selects a preferred physical slicing.

Locality

Solving an elliptic equation on each leaf is spatially nonlocal. STF already contains temporal nonlocality through its causal memory kernel; the carrier adds a distinct spatial nonlocality. The two must be kept conceptually and mathematically separate.

Causality

An elliptic constraint does not by itself define a propagating superluminal signal. Nevertheless, boundary dependence can affect local solutions across a leaf. A completed theory must specify its initial-boundary formulation and show that operational observables cannot be controlled to transmit acausally.

Degree-of-freedom count

The boundary-defined carrier has no independent local canonical pair. Its normal is a functional of the metric and boundary data. The intended local inventory is therefore

$$\boxed{2\ \text{tensor modes} + 1\ \text{internal STF scalar mode} + 0\ \text{independent universal-clock waves.}}$$

This count is a construction target, not yet a completed Hamiltonian proof for the nonlocal action.

19. Global existence and uniqueness

CMC foliations do not exist in every globally hyperbolic spacetime. Some spacetimes admit no CMC Cauchy surface; others contain a CMC slice without being covered globally by CMC leaves. STF therefore must prove a theorem about its own admissible solution class rather than about all Lorentzian geometries.

A sufficient carrier theorem would assume:

1. a globally hyperbolic post-closure domain with suitable compact or cell-wise Cauchy surfaces;
2. a regular projected initial CMC leaf;
3. a compatible terminal macroboundary;
4. existence and uniqueness of a CMC foliation between them;
5. a positive finite lapse;
6. finite total or cell-normalized four-volume.

Under these hypotheses, accumulated four-volume defines a smooth ordering and the normalized leaf normal supplies the universal carrier.

The hypotheses are substantive. Their derivation from STF closure remains a theorem program.

20. Relation to measured and observed time

The carrier resolves ordering, not the entire measurement problem.

Universal time supplies:

- which events lie on the same selected leaf;
- which direction is future;
- the normal along which curvature evolution is evaluated;
- a global ordering between closure boundaries.

Internal clocks supply:

- cyclic phase recurrence;

- local records;
- operational durations and frequencies;
- the apparatus through which the universal history is measured.

The numerical label $T_U = f(v)$ remains invisible to an action depending only on the normalized normal. A relative rate between universal ordering and an operational clock requires a constitutive measurement law. The carrier paper does not assume that law.

21. Claim and dependency ledger

Claim	Status
an oscillating amplitude cannot be a global normalized-gradient clock	theorem
STF requires distinct universal ordering and internal phase	theorem consequence
first closure must be represented by a spacelike boundary	derived structural requirement
boundary Lorentzian distance gives a local clock	theorem before cut locus
the unit-eikonal clock is a generic global carrier	false
the parent Kretschmann scalar supplies an FLRW seed	derived
the curvature seed should be inserted as the bulk local clock	disfavored by higher-derivative audit
fixed-four-volume extremization gives CMC	theorem
the CMC lapse is elliptic	standard geometric result
exact FLRW seed and CMC normals agree	verified
a generic threshold seed is CMC	false
ordinary GR matter closes the seed–release junction	false for every $k \neq 0$
canonical ϕ automatically cancels the mismatch	false as an identity
the local surface-volume action is the cuscuton/CMC action	derived correspondence
minimal local cuscuton plus the STF rate preserves exact CMC	false generically
Bel–Robinson observer dependence repairs the linear mismatch	false on $R_0 \neq 0$ FLRW
radiation-origin norm has a standard linearization	false
a boundary-defined CMC normal adds a local clock wave	false by construction
the boundary-defined carrier is globally available on all STF solutions	open
its nonlocal metric variation is complete	open
the full causal kernel preserves the carrier constraints	open
the 10D parent derives the CMC-volume principle	open

22. Falsifiers and decisive tests

The candidate carrier fails if any of the following is established for the STF solution class.

1. The curvature seed cannot be mapped to a regular CMC representative without destroying closure or orientation.
2. The relevant post-closure spacetime admits no CMC foliation connecting the two boundaries.
3. More than one inequivalent branch satisfies the same complete boundary data with no selection rule.
4. The CMC lapse reaches zero, diverges, or changes sign.
5. The nonlocal metric response violates the diffeomorphism Ward identity or makes the initial-boundary problem inconsistent.
6. The resulting preferred foliation violates empirical preferred-frame or gravitational-wave constraints.
7. Coupling the causal response kernel introduces an additional unstable mode.
8. The radiation norm cannot be regularized from the parent without adding an arbitrary scale or changing the claimed response channel.

23. Completion program

The derivations reduce the universal-clock program to four calculations.

23.1 Boundary functional derivative

Compute $\delta N_{\mathrm{CMC}}[g]/\delta g$ beyond scalar FLRW perturbations, including lapse, shift, and terminal-boundary variations.

23.2 Ward/Bianchi closure

Insert the indirect normal variation into the complete metric equations and verify covariant stress conservation with the boundary terms included.

23.3 Causal memory coupling

Replace the local rate operator by the zero-mode-subtracted retarded kernel and derive its response on the selected CMC history. The memory state must not become an independent universal clock.

23.4 Existence theorem for the STF solution class

Prove that the closure-selected cosmological domain belongs to a CMC-foliable class, or state the restricted domain on which the effective theory is valid.

24. Conclusion

The two-clock theorem does not require STF to bolt a second propagating scalar onto the four-dimensional action. It requires a distinct carrier of universal ordering. The derivation chain developed here points to a geometric carrier:

$$\boxed{\text{parent-curvature seed} \longrightarrow \text{finite fixed-volume projection} \longrightarrow \text{boundary-selected CMC foliation} \longrightarrow N^{\mu}_{\mathrm{CMC}}[g].}$$

The local cuscuton action proves that surface volume and CMC are field-theory neighbors, but its coupling obstruction shows why the universal carrier should not automatically be promoted to another local matter field. Universal time is instead most economically represented as a collective property of the globally closed geometry.

The internal scalar continues to oscillate. Its phase supplies recurrence. The CMC-volume foliation supplies ordering. Neither replaces the other.

The result is conditional rather than complete: global existence, nonlocal metric variation, causal memory, and radiation differentiability remain open. But the carrier question has been reduced from an unrestricted khronon fork to a specific geometric theorem program with explicit equations and falsifiers.

Acknowledgements and verification

The obstruction and construction results were developed in adversarial external sessions (August 2026) and the algebraic results are covered by the accompanying reproducibility script. Before publication, the paper's two central quantitative results — the §9 junction functional and the §14 compatibility obstruction — were additionally re-derived independently from a full perturbed-Riemann computation on matter FLRW, reproducing $(10/9)y + (8/81)y^2$, $\pi_{S,k}$, $\pi_{\mathrm{CMC},k}$ and $y(2y + 15) = 0$ exactly.

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Reproducibility

The accompanying script `the_universal_clock_carrier_checks.py` verifies the central algebraic results used in this paper:

- the complex-modulus normalization $\tau = e^{4\sigma}$;
- shift-charge dilution;
- the FLRW curvature-seed scaling;
- fixed-four-volume CMC stationarity;
- the matter Seed–Release Junction Functional;
- the CMC inverse-elliptic response;
- Bel–Robinson perturbative order counting;
- local coupling backreaction;
- the CMC/uniform-curvature incompatibility;
- and radiation-origin nondifferentiability.

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