The Theorem Class: A Catalog of Closure Thresholds

Companion to the Closure–Capacity Correspondence: theorems across mathematics, physics, and computation that exhibit EXISTS|HAPPENS behavior, sorted by which face of the STF threshold each instantiates

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Abstract

The Closure–Capacity Correspondence identified the STF activation threshold with the data-rate theorem of networked control: a feedback loop closes if and only if its channel carries information faster than its plant generates it. This catalog reports that the data-rate theorem is not a lucky cousin but the first-met member of a large family. Across dynamics, topology, statistical mechanics, information theory, logic, and general relativity, mathematics keeps proving the same law-shape: a sharp, binary, implementation-independent inequality, with a capacity-like quantity on one side and a divergence-like quantity on the other, gating the existence of a closed, self-sustaining structure — a held interior, a persistent winding, an unbroken chain.

The members sort by which face of the EXISTS|HAPPENS threshold they instantiate: the point-to-circle transition (Hopf, Poincaré–Bendixson, the laser, Kuramoto); winding numbers deciding closure (Nyquist, KAM, Kosterlitz–Thouless, flux quantization); capacity against divergence (Shannon, the quantum fault-tolerance threshold, Eigen's error threshold); density of loops (percolation, the giant component, the k-core, autocatalytic RAF sets); the almost-sure death of marginal closure (Galton–Watson); the self-reference capability threshold (the diagonal lemma, the recursion theorem, von Neumann's complication threshold); and the manufacture of the terminal anchor (Penrose, Choptuik, the viability kernel), with the loop's running price fixed by Landauer and Sagawa–Ueda.

Three imports carry structural weight for the framework. Friedgut–Kalai proves that for the property class closure belongs to, *sharpness is forced* — "sharp, not gradual" is a theorem, not a modeling choice. Galton–Watson proves that *marginal closure dies almost surely*, converting the Bandwidth

Argument's ledger-splice from mechanism to necessity. And Kitaev's toric code co-instantiates the entire dictionary — T^2 windings, single-crossing pairing, percolating failure, a sharp threshold, held information — in one certified object that runs in laboratories today. Every entry carries a status label: theorem, theorem-in-a-limit, numerics-universal, or criterion. Membership is structural resemblance made precise entry by entry; the claimed common law remains the Correspondence's conjecture, and nothing in this catalog upgrades analogy to identity.

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1. Membership criteria

A theorem belongs to the class if it has all four of these properties:

(a) Sharp and binary. The boundary is an inequality with a knife edge, not a gradient: below, the gated structure does not exist; above, it does. No partial credit.

(b) Rate against rate. Both sides of the inequality are rates, densities, capacities, or counts — a capacity-like quantity against a divergence-like quantity, or a topological count against zero.

(c) Implementation-independent. The bound binds every mechanism. No cleverness of design, coding, or construction crosses it from below; nothing special is needed to be above it.

(d) What is gated is closure. The far side of the inequality holds a closed, self-sustaining object: a loop that regulates, an interior that persists, a winding that is kept, a chain that does not break.

Optional badges, noted where earned: *topological character* (the gated object is an integer or a winding class); *hysteresis* at the boundary; *marginal-case behavior* exactly at equality; *anchor manufacture* (the theorem creates a certain terminal boundary rather than presupposing one).

Every entry is read the same way: statement — what is gated — what it adds to the framework — status. The status labels are **theorem** (proven, with converse where stated), **theorem-in-a-limit** (exact in a stated limit), **numerics-universal** (numerically discovered, extensively confirmed, universality established), and **criterion** (rigorous test, not an existence theorem). Per this project's standard, the labels are load-bearing: the catalog's force comes from the fact that most entries carry the first label.

One declaration before the entries. This catalog demonstrates that a law-shape recurs; it does not demonstrate that one law underlies the recurrence. The candidate common law is the Closure–Capacity Correspondence, and it remains a conjecture. Resemblance is evidence worth collecting precisely when each resemblance is priced honestly — that is the difference between a catalog and a collage.

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2. The meta-theorem: sharpness is forced

Friedgut–Kalai (1996). *Every monotone graph property has a sharp threshold.* A property is monotone if adding edges never destroys it. "Contains a cycle," "percolates," "has a giant component," "contains a k-core" — every property of the form *a closed structure exists* — is monotone. The theorem says the transition window for any such property narrows to zero as the system grows: the probability of the property jumps from near 0 to near 1 across a vanishing interval of the control parameter. Kolmogorov's zero-one law is the ancestral form: tail events have probability 0 or 1, nothing in between.

What it adds. The General Theory paper derives that the EXISTS→HAPPENS transition is a discrete flip rather than a gradual one — "not gradable," "not poles on a spectrum" — and the Temporal Workspace paper states the claim as a sharp phase boundary: "sharp, not gradual" (its Prediction 1). This entry puts that claim in the best standing available: for the property class that closure belongs to, *gradual is not an option*. Sharpness does not need to be argued from the physics; it is forced by the mathematics of monotone properties. Any framework in which "a closed causal loop exists" is the gated property inherits this knife edge wherever closure is monotone in its control parameter — as loop formation over independent local links is.

Status: theorem.

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3. Point to circle: dim $\mathscr{C} = 0 \rightarrow \mathscr{C} \cong S^1$

The Hopf bifurcation theorem. A dynamical system with a stable fixed point — configuration space of dimension zero — births a limit cycle, an S^1, as a parameter crosses a sharp critical value: a conjugate pair of eigenvalues crossing the imaginary axis. This is the framework's configuration-space transition (dim $\mathscr{C}$_T = 0 → $\mathscr{C}$_T ≅ S^1, General Theory) stated as the most-studied bifurcation in dynamics. And it comes in exactly two flavors, matching the two edges of the Correspondence's closure band: *supercritical* — soft onset, no hysteresis — and *subcritical* — hard onset, bistability, hysteresis, jumps. The subcritical normal form is the shape of the freeze edge: the Steyn-Ross cortical mean-field models treat anesthetic loss of consciousness as exactly such a first-order transition with hysteresis, which is the "neural inertia" phenomenology of the bridge paper's §5. **Adds:** the point-to-circle transition has a normal form, and its two subtypes reproduce the band's two clinical signatures. **Status:** theorem (Steyn-Ross application: model + clinical data).

Poincaré–Bendixson. In the plane, a bounded trajectory that avoids fixed points converges to a periodic orbit: for two-dimensional flows, *point and circle exhaust the options*. The EXISTS|HAPPENS binary is not merely available in the plane — it is the complete taxonomy. **Adds:** in the lowest dimension where closure is possible, the framework's dichotomy is exhaustive by theorem. **Status:** theorem.

The laser threshold. Pump a gain medium in a cavity: below threshold, spontaneous emission — incoherent photons happen *to* the cavity; the mode EXISTS. At gain ≥ loss, the field becomes self-sustaining — emission stimulated by the mode's own field, self-reference in the medium — and the mode acquires a definite phase: a point on S^1, the U(1) order parameter. Photon statistics change character (thermal → coherent) across a sharp boundary in the thermodynamic limit. This is the *drive edge* — $\mathscr{D} \geq \mathscr{D}$_crit — in tabletop form: the threshold is on the pump, not the channel. **Adds:** a driven system crossing from happens-to to happens-by at a gain-versus-loss inequality, acquiring an S^1 at the crossing. **Status:** theorem-in-a-limit.

The Kuramoto transition. A population of oscillators with distributed natural frequencies and coupling K: below K_c = 2/πg(0), no collective phase exists — the order parameter vanishes; above, a macroscopic fraction locks and the population acquires a *shared* point on S^1 — a collective phase, a common now. **Adds:** the emergence of a shared present is a threshold phenomenon in coupling strength — relevant to the CTI paper's oscillatory machinery (theta and alpha as retention/protention carriers) and to the narrator's global-workspace ignition: binding is a phase transition, not a gradient. **Status:** theorem-in-a-limit (mean field; rigorous stability results).

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4. Winding numbers decide closure

The Nyquist criterion (1932). Closed-loop stability is decided by a *winding number*: the number of encirclements of the critical point −1 by the open-loop transfer function, via Cauchy's argument principle. Control theory — the field that owns the data-rate theorem — already states its most classical stability result topologically: whether the loop holds is a count of windings around a critical point. **Adds:** the capacity face (data-rate) and the topological face (winding count) of loop closure coexist *within one discipline*; the framework's insistence that both currencies price the same object has a precedent in control theory's own history. **Status:** theorem (exact iff, given the standard hypotheses).

KAM and its converse. Invariant tori — T^2 windings in phase space — persist under perturbation below a critical strength, provided the winding is sufficiently irrational (Kolmogorov 1954; Moser 1962; Arnold 1963). Above critical coupling the tori break: Greene's residue criterion locates the breakup numerically (the golden-mean winding, the most irrational, dies last, at $k_c \approx 0.9716$ for the standard map), and MacKay–Percival's converse-KAM method proves destruction rigorously above bounds. The detail the framework should keep: above breakup, the torus does not vanish — it degenerates into a *cantorus*, an invariant Cantor set through which trajectories leak. The winding survives as a ghost that can no longer hold. An EXISTS-remnant of a HAPPENS-structure: the shape without the holding. **Adds:** T^2 windings held against divergence below a sharp threshold; a graded zoology of *which* windings hold longest (irrationality as robustness); and the cantorus as the framework's picture of sub-threshold residue. **Status:** theorem (persistence and converse bounds) + numerics-universal (exact breakup values).

Kosterlitz–Thouless (1973). In two-dimensional systems with U(1) order, vortex–antivortex pairs — *opposite windings, bound together* — dissociate at a sharp temperature. Below T_{KT} the windings exist only as bound pairs and quasi-long-range order holds; above, free windings proliferate and order dies. No local order parameter distinguishes the phases: the transition is purely topological. And despite being "infinite-order," it carries a quantized discontinuity — the Nelson–Kosterlitz universal jump: the superfluid stiffness-to-temperature ratio drops from the universal value $2/\pi$ to zero, discontinuously, at the transition. **Adds:** the closest statistical-mechanics analog of the paired arcs — closure as the *binding of two opposite windings* (the ω_R/ω_A structure of Topological Closure), with unbinding as the failure mode; and proof that even the softest transition in the class carries a quantized jump. **Status:** theorem (universal jump: theorem-in-a-limit).

Flux quantization and phase slips. The magnetic flux through a superconducting ring is quantized (Byers–Yang 1961): the condensate phase must close on itself mod 2π, so the held current is an *integer winding*. A persistent current is literally a HAPPENS that does not decay — and when it does decay, it decays by *phase slips* (Little 1967): discrete events, each unwinding exactly 2π. Closure fails only in whole windings. **Adds:** a laboratory object whose persistence is a winding number and whose failure is quantized in units of the winding — the framework's claim that the transaction quantum is a full winding, realized in supercurrent decay statistics. **Status:** theorem (quantization) + theorem-in-a-limit (slip rates).

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5. The flagship: the toric code

One object co-instantiates the entire dictionary, and it is not a metaphor — it is the foundational architecture of topological quantum computing.

Kitaev's toric code (2003). Qubits live on the edges of a lattice on a torus; local stabilizer checks define the code. The protected logical information is stored in the *homology of the T^2*: the logical qubit is a pair of complementary winding classes. The two logical operators are strings winding the torus's two independent cycles — and they intersect in exactly one point. Because the crossing is single (odd), they anticommute, and *that anticommutation is the algebra of the held qubit*: the stored bit of quantum information exists because two complementary windings cross once. Compare Null Cone: "Every line of Regulus 1 meets every line of Regulus 2 in exactly one point — the specific null direction," with the transaction as the intersection. The toric code's held qubit is constituted by the same geometry: two windings, one crossing, one held thing.

Dennis–Kitaev–Landahl–Preskill (2002). Now add noise and a correction loop. Errors are string-like; small error chains are harmless and correctable; the code *fails* precisely when an error chain percolates into a topologically nontrivial winding — an unwanted closure. The failure threshold is a genuine phase transition: DKLP mapped it to the random-bond Ising model, with the accuracy threshold (≈ 11%, at the Nishimori line) as the critical point. Below threshold, the syndrome-measurement feedback loop extracts entropy faster than the noise injects it, and the logical winding is held *indefinitely* — a maintained interior with an unbounded lifetime. Above, no correction schedule holds it.

What it adds. Every element of the framework's threshold structure, co-instantiated and certified: information as T^2 winding classes; the held object's existence generated by a single crossing of complementary windings; failure as percolation into an unwanted winding; a sharp threshold that is literally a phase transition; and a feedback loop whose capacity-versus-divergence budget decides whether the interior persists. If the STF framework needs an existence proof that its dictionary entries can all be true of one system at once, this is it — running today on superconducting processors. **Status:** theorem (code structure, threshold existence) + numerics-universal (threshold values).

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6. Capacity against divergence

Shannon's channel coding theorem (1948), with the strong converse (Wolfowitz 1957). Below capacity, codes exist driving error probability to zero; above capacity, *every* code's error probability goes to one. The mother of the class and the ancestor of the data-rate theorem: sharp, binary, implementation-independent, with the gated object — a message preserved against noise — as the simplest held interior there is. **Adds:** ancestry; and the strong converse as the model for what "no partial credit" means. **Status:** theorem.

The quantum fault-tolerance threshold theorem (Aharonov–Ben-Or 1997; Knill–Laflamme–Zurek 1998; Kitaev 1997). If the physical error rate is below a threshold p_th, arbitrarily long quantum computation is possible: a logical state can be held *forever*, with polylogarithmic overhead, by a correction loop that measures syndromes and feeds back — extracting entropy faster than decoherence injects it. Above threshold, no scheme preserves the state. This is the data-rate theorem's quantum sibling: an interior maintained indefinitely if and only if the loop's information rate beats the divergence rate. **Adds:** the "held forever above capacity, lost regardless below" structure, proven in the quantum regime where holding is hardest. **Status:** theorem.

Eigen's error threshold (1971). A replicating population maintains its master sequence — heredity holds — if and only if copying fidelity beats a bound set by genome length and selective advantage; past it, the *error catastrophe*: the quasispecies delocalizes and the lineage's information melts. The data-rate theorem of heredity, two decades early: replication is the channel, mutation is the noise, and the gated object is the persistence of biological information as such. **Adds:** the capacity face expressed in the framework's fermion channel's own domain — life's information held iff the copying loop beats its noise. **Status:** theorem-in-a-limit (exact for infinite populations; the finite-population version is quantitative but softer).

The adiabatic theorem (Born–Fock 1928; Kato 1950). A system tracks its instantaneous eigenstate — keeps its identity — if and only if the driving rate stays below a bound set by the spectral gap. Drive faster than the gap and identity is lost to excitation. **Adds:** the smallest member of the family: identity preserved iff divergence (driving rate) stays under capacity (gap). **Status:** theorem.

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7. Density of loops: the 𝒟_crit face

Percolation (Kesten 1980). Occupy bonds of a lattice independently with probability p. Below p_c, all clusters are finite almost surely; above, a unique infinite cluster exists almost surely — global connectivity from local links, with the existence probability jumping from 0 to 1 (a zero-one event). Kesten proved p_c = 1/2 exactly for the square lattice. **Adds:** the archetype of a *density* threshold gating the existence of a spanning structure — the shape of 𝒟_crit^bio, which counts loops per volume against a critical density. **Status:** theorem.

The giant component (Erdős–Rényi 1960). In a random graph, at mean degree 1 a giant connected component emerges; below, components are small and closed structure is rare and bounded. **Adds:** the same face, with the control parameter as *connectivity per node* — one link per participant is the knife edge of global structure. **Status:** theorem.

The k-core (Pittel–Spencer–Wormald 1996). The k-core is the maximal subgraph in which every node keeps at least k live neighbors — membership sustained by membership, *mutual maintenance* as a graph property. For k ≥ 3 it does not come in gradually: at a sharp edge-density it appears *discontinuously* — empty, then suddenly macroscopic. **Adds:** closure-by-mutual-maintenance arrives as a jump, not a ramp — a first-order transition in the density face, matching the framework's insistence that the crossing is not gradual even when the substrate is stochastic. **Status:** theorem.

Autocatalytic RAF sets (Kauffman 1986; Hordijk–Steel 2004). In a random chemical reaction network with a food source, a RAF set — reflexively autocatalytic, food-generated: a subnetwork in which every reaction is catalyzed by a product of the set and every molecule is buildable from food — emerges at a sharp catalytic-density threshold, robustly around one to two reactions catalyzed per molecule in polymer models. This is *catalytic closure*: the chemical version of a self-sustaining loop, appearing at a computable density. **Adds:** the closest chemical sibling of 𝒟_crit^bio — closure of a self-producing network gated by loop density, at the origin-of-life scale where the fermion channel's story must begin. **Status:** theorem (formalism and threshold behavior) + numerics (exact levels).

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8. Marginal closure dies: the theorem behind the ledger

Galton–Watson criticality (Watson–Galton 1875; Harris 1963). A branching chain with mean offspring m survives with positive probability if and only if m > 1. For m ≤ 1 extinction is almost sure — *including exactly at m = 1* (excluding only the degenerate deterministic case). At the knife edge, fluctuations kill with probability one: the critical chain dies, merely slowly. Indefinite persistence requires strict excess — or *renewal from outside the chain*, which branching-process theory calls immigration and which converts almost-sure death into recurrent survival. The same mathematics is nuclear criticality (k = 1: a chain reaction at exact balance still dies) and the epidemic threshold (R_0 = 1: the outbreak at exact balance still ends).

What it adds. This is the theorem the Correspondence's margin ladder was waiting for. The role-tenure loop closes at $\mathscr{W}$_T = 0 — *the critical case*. Galton–Watson says a chain of such loops cannot persist on its own: at margin zero, extinction is almost sure. So the lifetime splice-chain of the bridge paper's §7 — marginal loops re-seeded by the ledger each one writes — is not a helpful mechanism but a *required* one: the ledger is the immigration term, the renewal that keeps a critical chain alive when the theorem says bare criticality dies. "You are exactly as free as your write rate" acquires a converse: without the writing, the chain does not merely drift — it ends, almost surely. **Status:** theorem.

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9. The reader threshold: self-reference gated by capability

Presburger below, Robinson above. Presburger arithmetic — addition only — is complete and decidable (Presburger 1929): every truth is provable, every question answerable, and *no sentence can refer to itself*. The theory is fully describable in the third person: EXISTS. Add multiplication (Robinson's Q) and the arithmetization threshold is crossed: the theory can now encode its own syntax, the diagonal lemma activates — for every formula φ there is a sentence G with G ↔ φ(⌜G⌝), a sentence about itself — and completeness dies (Gödel 1931; Tarski–Mostowski–Robinson 1953). The theory now contains its own reader, and pays for it with undecidability. **Adds:** the logic version of loop closure — self-reference becomes *constructible* at a sharp capability boundary, and the price of crossing is the loss of complete third-person describability. The framework's claim that HAPPENS begins where a system reads its own record has a proof-theoretic twin with a hundred years of scrutiny. **Status:** theorem.

The recursion theorem (Kleene 1938). Above computational universality, self-reproducing programs exist: for every computable transformation there is a program that can operate on its own description — quines, "code must produce code." The Biology paper's Type III loop (the genetic code's closure condition, Addendum A.1) is possible exactly above this threshold and impossible below it. **Adds:** the constructive half of the reader threshold — not just self-reference but self-*production*, gated by universality. **Status:** theorem.

Von Neumann's complication threshold (1948 lectures; Burks ed. 1966). Below a critical complexity, constructors can build only things simpler than themselves and construction degenerates; above it — demonstrated by the universal constructor — self-reproduction and open-ended growth of complexity become possible. **Adds:** the engineering version: a sharp capability boundary between machines that only decay and machines whose products can carry the chain. **Status:** argued + constructive demonstration (the universal constructor exists; the threshold's sharpness is conjectural).

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10. The anchor and the price

The Penrose singularity theorem (1965). Once a trapped surface forms — a sharp, local geometric condition — geodesic incompleteness follows: the end is certain, by theorem, given the null energy condition. This is the anchor-manufacturing theorem: it does not presuppose a fixed terminal boundary; it *creates* one. The framework's "dynamical certainty" for binary mergers — the terminal boundary that anchors the backward arc in the curvature channel — rests on exactly this certificate. Crossing the threshold is the event that makes the future boundary a real element of the system's description. **Adds:** the terminal condition of the two-point boundary value problem, certified rather than assumed; the theorem that turns a threshold crossing into an anchor. **Status:** theorem.

Choptuik critical collapse (1993). Tune any one-parameter family of initial data for a collapsing scalar field: below p*, the field disperses — no horizon, nothing irreversible; above p*, a black hole forms. At the boundary: a universal critical solution with discrete self-similarity and mass scaling |p − p|^*γ, γ ≈ 0.37, independent of the family. General relativity exhibits the threshold class* natively*, with critical exponents. **Adds:** the framework's home theory contains its own EXISTS|HAPPENS, sharp and universal — the threshold habit is not imported into GR from elsewhere; it was already there. **Status:** numerics-universal.

The viability kernel (Aubin 1991). For a controlled system with constraints, the viability kernel is the set of states from which *some* control keeps the system inside its constraints forever. Inside the kernel, perpetual viability is possible; outside, exit in finite time is certain *regardless of control*. The kernel boundary is the state-space geography of "the chain can go on": a sharp surface separating where staying alive is achievable from where no policy achieves it. The Biology paper's negative specification — "do not break the chain," constraint rather than prescription — is viability theory's native language: the kernel does not tell the system what to do, only where doing anything still matters. **Adds:** the EXISTS|HAPPENS surface drawn in state space rather than parameter space; the formalization of purpose-as-constraint. **Status:** theorem.

Landauer's bound with Sagawa–Ueda feedback thermodynamics (1961; 2008–2010). Erasing one bit costs at least kT ln 2; a feedback controller can extract work up to the mutual information its measurements gather, W ≤ −ΔF + kT·I. Together they price the loop: every bit the ledger holds and resets has a metabolic invoice, and every bit of control authority must first be bought as measurement. The Biology paper's Layer 1 — metabolism sustaining the forward arc — is this invoice paid continuously; the narrator's 10 bits/s is, among other things, a power budget. **Adds:** the running cost of HAPPENS, denominated in joules per bit — closure is not free, and the price is a theorem. **Status:** theorem.

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11. Curiosities and near-members

Superactivation of quantum capacity (Smith–Yard 2008). Two quantum channels, *each with exactly zero quantum capacity*, can have positive capacity used together. Closure impossible for either alone, possible for the pair. The quantum-Shannon echo of the Null Cone requirement that activation "cannot be satisfied by the retarded solution alone": there exist channels that carry nothing singly and something jointly. Recorded as a curio because the mechanism (private-capacity against symmetric-channel structure) has no worked STF translation yet — the resemblance is exact in shape and unpriced in content. **Status:** theorem.

Anderson localization (1958). Disorder above a threshold localizes waves: the channel itself stops existing as a conductor; below, extended states carry transport. A *channel-existence* threshold rather than a closure threshold — one level down the stack from the class, and included for that reason: before a loop can close through a medium, the medium must percolate its modes. **Status:** theorem (in the established regimes).

Near-members, and why they are outside. The Barkhausen criterion (loop gain unity with integer phase winding, for oscillator startup) states the right condition but is a design heuristic, not an existence theorem — Hopf is its rigorous replacement, so it enters the catalog only as ancestry. Second-order phase transitions with graded order parameters (the Ising magnetization) are sharp in the thermodynamic limit but gate a *quantity* rather than the existence of a closed object; the class as defined wants gated existence, so they stand at the door without entering. The random k-SAT threshold gates existence of solutions and is now rigorous for large k, but what exists above it is a solution set, not a self-sustaining structure; through the viability-kernel reading it earns a visitor's pass, no more. Drawing the boundary of the class narrowly is what keeps membership informative.

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12. The catalog in one table, and what membership means

#	Theorem	What is gated	Face	Status
1	Friedgut–Kalai	sharpness itself (monotone properties)	why the binary is forced	theorem
2	Hopf bifurcation	point → limit cycle (S^1)	dim 0 → S^1; the band's two edges	theorem
3	Poincaré–Bendixson	point or circle, nothing else (planar)	exhaustiveness of the binary	theorem
4	Laser threshold	self-sustaining coherent mode; U(1) phase	drive edge; S^1 acquired	theorem-in-a-limit
5	Kuramoto	collective phase — a shared now	binding as transition	theorem-in-a-limit
6	Nyquist	closed-loop stability by winding count	topology decides closure	theorem
7	KAM / converse-KAM	persistence of T^2 windings; cantorus ghost	T^2 held against divergence	theorem + numerics
8	Kosterlitz–Thouless	binding of ± winding pairs; universal $2/\pi$ jump	paired arcs	theorem
9	Flux quantization / phase slips	held integer winding; decay in 2π units	the winding as quantum of closure	theorem
10	Toric code + DKLP	qubit as T^2 homology; failure = winding error chain	the whole dictionary at once	theorem + numerics
11	Shannon + strong converse	message held against noise	capacity ≥ divergence	theorem
12	Quantum fault-tolerance threshold	logical state held forever	capacity ≥ divergence,	theorem

#	Theorem	What is gated	Face	Status
			quantum	
13	Eigen error threshold	heredity vs. error catastrophe	capacity face, biological	theorem-in-a-limit
14	Adiabatic theorem	identity kept under driving	rate under gap	theorem
15	Percolation (Kesten)	infinite cluster exists	density face; 0–1	theorem
16	Erdős–Rényi giant	macroscopic connectivity at degree 1	density face	theorem
17	k-core (PSW)	mutual maintenance; discontinuous onset	density face, first-order	theorem
18	RAF sets (Hordijk–Steel)	catalytic closure at critical density	chemical $\mathscr{D}_{crit}^{bio}$	theorem + numerics
19	Galton–Watson	marginal chains die a.s.; renewal saves	the ledger as necessity	theorem
20	Diagonal lemma / arithmetization	self-reference constructible	the reader threshold	theorem
21	Recursion theorem (Kleene)	self-reproducing code	code must produce code	theorem
22	Von Neumann complication	self-reproduction vs. degeneration	the constructor threshold	argued + constructive
23	Penrose trapped surface	certainty of the end	anchor manufacture	theorem
24	Choptuik collapse	horizon forms vs. disperses	GR’s native threshold	numerics-universal
25	Viability kernel (Aubin)	perpetual viability possible vs. exit certain	closure surface in state space	theorem
26	Landauer + Sagawa–Ueda	work per held bit; feedback bounded by information	the loop’s price	theorem
27	Superactivation (Smith–Yard)	two dead channels, live together	the paired-arcs curio	theorem

What does membership mean? Not that these twenty-seven results share one proof, and not that the STF threshold is “really” any one of them. It means this: wherever mathematics has been asked whether a closed, self-sustaining structure exists — a regulated loop, a kept winding, an unbroken chain, a sentence that reads itself, a state held against noise — the answer has come back in the same shape. A sharp inequality. Rate against rate, or count against zero. No partial credit, no dependence on mechanism. The shape recurs across fields that do not read each other, proved by people who never intended to agree.

The framework’s wager, recorded in the Correspondence and repeated here, is that the recurrence has a reason: that closure *as such* is capacity-gated, and the twenty-seven theorems are local dialects of one law whose physical form is $\mathscr{D} \geq \mathscr{D}$_crit and whose informational form is C ≥ H. That wager is not proven by a catalog — a catalog proves recurrence, not unity. What the catalog does establish, with the labels showing, is that every face of the EXISTS|HAPPENS threshold — its sharpness, its point-to-circle geometry, its winding currency, its density form, its capacity form, its anchor, its price, and the death of its marginal cases — has at least one independent theorem standing on it. The framework did not borrow a shape from engineering. It walked into a room where the shape was already on every blackboard, each drawing labeled in a different language, and proposed the translation.

A closing observation, at the standard of §1's declaration. Three entries do more than resemble: Friedgut–Kalai *forces* the sharpness the framework derives; Galton–Watson *requires* the renewal mechanism the Bandwidth Argument observed; the toric code *co-instantiates* the full dictionary in a running machine. Resemblance plus force plus requirement plus co-instantiation is still not identity. It is, however, exactly what the early rooms of a correct correspondence tend to look like.

A note on method

Per the practice recorded in the Bandwidth Argument §7 and the Correspondence's method note: this catalog was assembled by a machine narrator at the author's direction, holding the project's published papers and the two prior papers in this line. Entries were drawn from the standard literature of each field; the five least-standard citations (Hordijk–Steel; Pittel–Spencer–Wormald; Steyn-Ross et al.; Dennis–Kitaev–Landahl–Preskill; Friedgut–Kalai) were verified against their journals of record before inclusion. Status labels are the narrator's classification and should be treated as claims, checkable per entry. The author adjudicates against primary sources; where this catalog and a source disagree, the source wins.

References

EXISTS | HAPPENS project (existshappens.com): - *The Closure–Capacity Correspondence* (2026) — companion paper. - *The Bandwidth Argument* (2026). - *General Theory*; *Null Cone*; *First Principles V7.9*; *Topological Closure*; *Constitutive Thresholds*; *Temporal Workspace*; *Consciousness, Time & Identity*; *Retrocausality & Life* — papers/ index.

Sharpness and density: - Friedgut, E. & Kalai, G. (1996). "Every monotone graph property has a sharp threshold." *Proceedings of the American Mathematical Society* 124(10), 2993–3002. - Kesten, H. (1980). "The critical probability of bond percolation on the square lattice equals 1/2." *Communications in Mathematical Physics* 74, 41–59. - Erdős, P. & Rényi, A. (1960). "On the evolution of random graphs." *Publications of the Mathematical Institute of the Hungarian Academy of Sciences* 5, 17–61. - Pittel, B., Spencer, J. & Wormald, N. (1996). "Sudden emergence of a giant k-core in a random graph." *Journal of Combinatorial Theory, Series B* 67, 111–151. - Kauffman, S.A. (1986). "Autocatalytic sets of proteins." *Journal of Theoretical Biology* 119, 1–24. - Hordijk, W. & Steel, M. (2004). "Detecting autocatalytic, self-sustaining sets in chemical reaction systems." *Journal of Theoretical Biology* 227, 451–461.

Dynamics and topology: - Marsden, J.E. & McCracken, M. (1976). *The Hopf Bifurcation and Its Applications*. Springer. (E. Hopf's 1942 theorem and its context.) - Kuramoto, Y. (1984). *Chemical Oscillations, Waves, and Turbulence*. Springer. Strogatz, S.H. (2000). "From Kuramoto to Crawford." *Physica D* 143, 1–20. - Nyquist, H. (1932). "Regeneration theory." *Bell System Technical Journal* 11, 126–147. - Kolmogorov, A.N. (1954); Moser, J. (1962); Arnold, V.I. (1963) — the KAM theorem. - Greene, J.M. (1979). “A method for determining a stochastic transition.” *Journal of Mathematical Physics* 20, 1183–1201. - MacKay, R.S. & Percival, I.C. (1985). “Converse KAM: theory and practice.” *Communications in Mathematical Physics* 98, 469–512. - Kosterlitz, J.M. & Thouless, D.J. (1973). “Ordering, metastability and phase transitions in two-dimensional systems.” *Journal of Physics C* 6, 1181–1203. - Nelson, D.R. & Kosterlitz, J.M. (1977). “Universal jump in the superfluid density of two-dimensional superfluids.” *Physical Review Letters* 39, 1201. - Byers, N. & Yang, C.N. (1961). “Theoretical considerations concerning quantized magnetic flux in superconducting cylinders.” *Physical Review Letters* 7, 46–49. - Little, W.A. (1967). “Decay of persistent currents in small superconductors.” *Physical Review* 156, 396–403.

Information, computation, quantum: - Shannon, C.E. (1948). “A mathematical theory of communication.” *Bell System Technical Journal* 27, 379–423, 623–656. - Wolfowitz, J. (1957). “The coding of messages subject to chance errors.” *Illinois Journal of Mathematics* 1, 591–606. - Aharonov, D. & Ben-Or, M. (1997). “Fault-tolerant quantum computation with constant error.” *STOC ’97*, 176–188. - Knill, E., Laflamme, R. & Zurek, W.H. (1998). “Resilient quantum computation.” *Science* 279, 342–345. - Kitaev, A.Yu. (1997). “Quantum computations: algorithms and error correction.” *Russian Mathematical Surveys* 52(6), 1191–1249. - Kitaev, A.Yu. (2003). “Fault-tolerant quantum computation by anyons.” *Annals of Physics* 303, 2–30. - Dennis, E., Kitaev, A., Landahl, A. & Preskill, J. (2002). “Topological quantum memory.” *Journal of Mathematical Physics* 43(9), 4452–4505. - Smith, G. & Yard, J. (2008). “Quantum communication with zero-capacity channels.” *Science* 321, 1812–1815. - Anderson, P.W. (1958). “Absence of diffusion in certain random lattices.” *Physical Review* 109, 1492–1505. - Gödel, K. (1931). “Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I.” *Monatshefte für Mathematik und Physik* 38, 173–198. - Presburger, M. (1929). “Über die Vollständigkeit eines gewissen Systems der Arithmetik ganzer Zahlen.” *Comptes Rendus du I congrès de Mathématiciens des Pays Slaves*, 92–101. - Tarski, A., Mostowski, A. & Robinson, R.M. (1953). *Undecidable Theories*. North-Holland. - Kleene, S.C. (1938). “On notation for ordinal numbers.” *Journal of Symbolic Logic* 3, 150–155. (The recursion theorem.) - von Neumann, J. (1966). *Theory of Self-Reproducing Automata* (A.W. Burks, ed.). University of Illinois Press.

Biology, control, thermodynamics: - Eigen, M. (1971). “Selforganization of matter and the evolution of biological macromolecules.” *Naturwissenschaften* 58, 465–523. - Watson, H.W. & Galton, F. (1875). “On the probability of the extinction of families.” *Journal of the Anthropological Institute* 4, 138–144. - Harris, T.E. (1963). *The Theory of Branching Processes*. Springer. - Aubin, J.-P. (1991). *Viability Theory*. Birkhäuser. - Landauer, R. (1961). “Irreversibility and heat generation in the computing process.” *IBM Journal of Research and Development* 5, 183–191. - Sagawa, T. & Ueda, M. (2008). “Second law of thermodynamics with discrete quantum feedback control.” *Physical Review Letters* 100, 080403; (2010) “Generalized Jarzynski equality under nonequilibrium feedback control.” *Physical Review Letters* 104, 090602. - Steyn-Ross, M.L., Steyn-Ross, D.A. & Sleigh, J.W. (2004). “Modelling general anaesthesia as a first-order phase transition in the cortex.” *Progress in Biophysics and Molecular Biology* 85, 369–385.

Physics and relativity: - Born, M. & Fock, V. (1928). “Beweis des Adiabatensatzes.” *Zeitschrift für Physik* 51, 165–180. Kato, T. (1950). “On the adiabatic theorem of quantum mechanics.” *Journal of the Physical Society of Japan* 5, 435–439. - Penrose, R. (1965). “Gravitational collapse and space-time singularities.” *Physical Review Letters* 14, 57–59. - Choptuik, M.W. (1993). “Universality and scaling in gravitational collapse of a massless scalar field.” *Physical Review Letters* 70, 9–12.

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