Radiative Lepton Flavour Violation

MEG-II as a Probe of Generation Structure

Z. Paz · ORCID 0009-0003-1690-3669V0.12026

Abstract

The winding mode mechanism derived in Paper 3 of this series generates not only the Z→μτ branching ratio but also radiative lepton flavour violation (LFV) via the same Yukawa couplings. Since the lightest winding mode W̃ is electrically neutral, it cannot generate the photon dipole operator at one loop directly. Instead, the dominant radiative LFV contribution arises from the triangle diagram in which the photon attaches to the virtual charged lepton inside the loop — the standard magnetic dipole topology with one chirality flip.

For a scalar mediator W̃ with Yukawa couplings Y_phys, the branching ratios scale as:

$$\mathrm{BR}(\ell_i \to \ell_j \gamma) \propto \left|\frac{1}{16\pi^2 M_{\rm wind}^2} \sum_k Y_{ik} Y_{jk}^* m_k\right|^2$$

where the sum runs over intermediate lepton flavors k with mass m_k. The τ-mass insertion dominates: the key combination is $|Y_{\mu\tau} \cdot m_\tau \cdot Y_{\tau e}{}^*|$.

Under the minimal assumption that the abstract generation sections (A_1, A_2, A_3) from the Griffiths residue computation correspond to the (e, μ, τ) mass eigenstates, the predictions are:

$$\mathrm{BR}(\mu\to e\gamma) \approx 5.7 \times 10^{-8}, \qquad \mathrm{BR}(\tau\to\mu\gamma) \approx 6.2 \times 10^{-11}, \qquad \mathrm{BR}(\tau\to e\gamma) \approx 1.8 \times 10^{-11}$$

The τ rates pass current experimental bounds comfortably. The μ→eγ rate violates the MEG-II bound BR(μ→eγ) < 3.1×10^{-13} by a factor of ~2×10^5 under this assumption.

This is not a falsification — it is a constraint. The Griffiths residue computation provides Y^(0) in the abstract wavefunction basis (A_1, A_2, A_3), which is not the physical mass eigenstate basis. The MEG-II bound requires that in the mass eigenstate basis, $|(YM_\ell Y^\dagger)_{\mu e}|$ is suppressed by at least a factor of ~430 relative to its value under the minimal assumption. We identify three mechanisms that can provide this suppression — generation assignment from Z_{10} representation theory, Kähler normalisation via the true HYM bundle metric, and cancellation in the KK tower — and establish that the first two are the most physically

motivated. MEG-II is therefore a direct probe of the generation structure of the CICY #7447/Z_{10} compactification.

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1. Introduction

Papers 1–3 of this series established the Kähler geometry of CICY #7447/Z_{10}, derived the PMNS predictions, and identified the winding mode mechanism for $Z \to \mu\tau$. The same Yukawa matrix Y_phys that gives BR($Z \to \mu\tau$) = 3.0×10^{-8} also contributes to radiative LFV processes $\ell_i \to \ell_j \gamma$. This paper evaluates those contributions.

1.1 The Winding Mode is Electrically Neutral

The lightest winding mode $\tilde{W}$ in the CICY #7447/Z_{10} compactification is a gauge-singlet scalar under the SM gauge group (established in Paper 3, Section 3.1). In particular, Q_EM($\tilde{W}$) = 0. A neutral scalar does not couple to the photon at tree level and therefore cannot generate the photon dipole operator at one loop through a diagram where the photon attaches to the $\tilde{W}$ line.

The photon must instead attach to the charged internal lepton line in the loop. This is the standard magnetic dipole topology, and it generates the correct dipole operator with a chirality flip proportional to the intermediate lepton mass.

1.2 What This Paper Establishes

This paper has two results. The first is a formula for the radiative LFV rates in terms of the Yukawa matrix. The second is a constraint: the MEG-II bound on BR($\mu \to e\gamma$) requires a specific relationship between the Yukawa matrix elements in the physical mass eigenstate basis, which constrains the generation structure of the compactification.

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2. The Radiative LFV Formula

2.1 Diagram Structure

The relevant one-loop diagram for $\ell_i \to \ell_j \gamma$ proceeds as follows:

- External incoming lepton ℓ_i emits a virtual winding mode $\tilde{W}$ and becomes a virtual lepton ℓ_k
- The virtual lepton ℓ_k emits the external photon γ (attaching to the charged lepton line)

- The virtual lepton ℓ_k absorbs the W̃ and becomes the external outgoing lepton ℓ_j

The two W̃ vertices carry Yukawa couplings Y_ik and Y_jk respectively. The chirality flip required for the magnetic dipole is provided by the internal lepton mass m_k.

2.2 The Amplitude

In the heavy winding mode limit M_wind >> m_lepton (valid here: m_τ/M_wind = 0.019), the amplitude takes the standard form:

$$\mathcal{A}_R(\ell_i \to \ell_j \gamma) = \frac{e}{16\pi^2 M_{\rm wind}^2} \sum_k Y_{ik} Y_{jk}^* m_k f\!\left(\frac{m_k^2}{M_{\rm wind}^2}\right)$$

$$\mathcal{A}_L(\ell_i \to \ell_j \gamma) = \frac{e}{16\pi^2 M_{\rm wind}^2} \sum_k Y_{ik}^* Y_{jk} m_k f\!\left(\frac{m_k^2}{M_{\rm wind}^2}\right)$$

where the loop function in the $x \to 0$ limit is:

$$f(x) \to \frac{1}{6} \quad \text{for } x = m_k^2/M_{\rm wind}^2 \ll 1$$

The kinematic correction at $x = m_\tau{}^2/m_Z{}^2 \approx 3.8 \times 10^{-4}$ is entirely negligible.

The combination entering the amplitude can be written compactly as:

$\sum_k Y_{ik} Y_{jk}{}^* m_k = (Y M_\ell Y^\dagger)_{ij}$

where $M_\ell = \mathrm{diag}(m_e, m_\mu, m_\tau)$ is the diagonal lepton mass matrix in the mass eigenstate basis.

2.3 The Decay Rate and Branching Ratio

$$\Gamma(\ell_i \to \ell_j \gamma) = \frac{\alpha m_i^3}{4\pi} \left|\mathcal{A}_R\right|^2$$

$$\mathrm{BR}(\ell_i \to \ell_j \gamma) = \frac{\Gamma(\ell_i \to \ell_j \gamma)}{\Gamma_i^{\rm total}}$$

with $\alpha = 1/128$ at the m_Z scale and $M_{\rm wind} = m_Z = 91.19$ GeV (Paper 3).

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3. The Basis Problem

3.1 What the Griffiths Residue Computes

The Yukawa matrix $Y_{ij}^{(0)} = \mathrm{Res}[A_i \cdot A_j/(Q_1 Q_2)]$ is defined in the basis of abstract generation wavefunctions (A_1,A_2,A_3) — the three ρ_0-eigenvectors of the 32×32 g-matrix on $H^0(X,\mathcal{O}(1,...,1))$ modulo Q_1. These are NOT labeled by (e,μ,τ) a priori.

The physical mass eigenstate basis requires knowing which linear combinations of (A_1,A_2,A_3) correspond to the physical lepton generations. This was previously identified as the outstanding computation. However, Step 22 of the derivations archive establishes that A_1, A_2, A_3 already the Z₁₀-equivariant generation sections of $H^1(\tilde{X},\tilde{V})$, identified via the connecting homomorphism $\delta : H^0(A,\mathcal{O}(1,...,1)) \to H^1(A,V)$ which is an isomorphism when $H^*(A,B) = 0$.

The generation assignment is therefore determined: the null eigenvector $v \approx 0.29A_1 + 0.49A_2 - 0.82A_3$ aligns with the lightest Gram eigenvalue $\lambda_1 = 1.354$ to 99.7% (Step 21). This is the electron generation. No further cohomology computation is needed for the basis.

The 5-patch Yukawa matrix in the abstract basis is:

$$Y^{(0)}_{\rm phys} = \varepsilon_K \times Y^{(0)} = 0.12074 \times \begin{pmatrix} 0 & 0.1018 + 0.4238i & 0.0612 + 0.2549i \\ 0.1018 + 0.4238i & -0.7338 - 0.0522i & -0.4813 + 0.1147i \\ 0.0612 + 0.2549i & -0.4813 + 0.1147i & -0.3135 + 0.1569i \end{pmatrix}$$

with singular values $\sigma_1 = 0.154$, $\sigma_2 = 0.027$, $\sigma_3 = 0$ (structural zero confirmed in Paper 2).

3.2 The Minimal Assumption and its Failure

The simplest assumption — that (A_1,A_2,A_3) directly correspond to (e,μ,τ) in the mass eigenstate basis — gives the rates quoted in the Abstract. The μ→eγ violation stems from the dominant term in $(YM_\ell Y^\dagger)_{\mu e}$:

$(YM_\ell Y^\dagger)_{\mu e} \approx Y_{\mu\tau} \cdot m_\tau \cdot Y_{\tau e}{}^* = 0.0597 \times 1.777 \times 0.0317 \approx 3.4 \times 10^{-3}$ GeV

For MEG-II compliance this combination must satisfy:

$|(YM_\ell Y^\dagger)_{\mu e}| < 8.97 \times 10^{-6}$ GeV

The current value exceeds this by factor ~380. The minimal assumption fails for μ→eγ while passing for τ→μγ and τ→eγ.

3.3 Why τ→μγ and τ→eγ Pass Naturally

The τ rates are dominated by the diagonal coupling $Y_{\tau\tau} \cdot m_\tau$:

$(YM_\ell Y^\dagger)_{\tau\mu} \approx Y_{\tau\tau} \cdot m_\tau \cdot Y_{\tau\mu}{}^*$

This does not involve a cross-generation product spanning two different mass scales. With $|Y_{\tau\tau}| = 0.0423$ and $|Y_{\tau\mu}| = 0.0597$:

$|(YM_\ell Y^\dagger)_{\tau\mu}| \approx 0.0423 \times 1.777 \times 0.0597 \approx 5.1 \times 10^{-3}$ GeV

This is comparable to the μe combination, but the τ decay rate is suppressed by the phase space factor $(m_\mu/m_\tau)^3 \approx 2.1 \times 10^{-4}$, placing BR(τ→μγ) ~ 10^{-11} — well below the current bound.

The asymmetry is kinematic: μ→eγ involves the light μ mass cubed in the phase space, making the violation appear only when expressed as a branching ratio.

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4. Resolution Mechanisms

The MEG-II constraint requires that in the physical mass eigenstate basis:

$|(YM_\ell Y^\dagger)_{\mu e}| < 8.97 \times 10^{-6}$ GeV

Three mechanisms can provide the required suppression of ~380–430× relative to the minimal-assumption value.

4.1 Generation Assignment from Z_{10} Representation Theory

The physical interpretation of which abstract section A_i corresponds to which generation is determined by the Z_{10} representation theory. The key observation: the structural zero $\sigma_3 = 0$ means that one linear combination of (A_1, A_2, A_3) does not couple at tree level. This is the massless generation — the electron candidate.

The null eigenvector is $v \approx 0.29A_1 + 0.49A_2 - 0.82A_3$, predominantly in the A_3 direction. This means A_3 is the mostly-decoupled generation. If the correct identification is:

$A_3 \leftrightarrow e$ (massless at tree level)

then the physical Yukawa matrix in the (e,μ,τ) basis is the matrix Y with columns/rows permuted. The (μe) entry in this permuted basis involves Y_{12} and Y_{31} — entries that may be substantially smaller.

This identification is now confirmed: The connecting homomorphism argument (Step 22) establishes that A_3 is the electron generation and the abstract sections are the physical generation basis. The electron wavefunction is predominantly A_3 (null eigenvector is 82% in the A_3 direction); the muon and tau are combinations of A_1 and A_2. The Donaldson Gram matrix confirms this: the null direction aligns with the lightest Gram eigenvalue $\lambda_1 = 1.354$ to 99.7%, independently confirming the electron identification. The generation assignment is resolved. What remains is the Yang-Mills PDE for the fibre metric $h_V(x)$ — a PDE on X that gives the true wavefunction norms.

4.2 Kähler Normalisation via the True HYM Metric

The Yukawa matrix used here employs the Fubini-Study approximation to the HYM bundle metric. The Donaldson balanced metric algorithm has now been run on CICY #7447/Z_{10} (N=20,000 points, 50 iterations, converged stably). The results definitively rule out the HYM metric as the resolution mechanism:

METHOD	Σ_1/Σ_2	NOTES
FS diagonal α=2	4.06	Previous estimate
Donaldson diagonal	5.79	Converged, stable
Donaldson full Gram $G^{-1/2}YG^{-1/2}$	2.71	Off-diag $G_{23}/\sqrt{(G_{22}G_{33})}$ = 0.84
Target m_τ/m_μ	16.81	PDG

None of the metric corrections approach the required ratio. The full Gram matrix shows that sections A_2 and A_3 are 84% correlated under the HYM inner product. The Z_{10} equivariant basis computation from the ambient space H^0(A, O(1,…,1)) has been exhausted and confirmed (Step 22–23): the abstract sections A_1, A_2, A_3 are already the equivariant generation sections; the T-operator converges to the same Gram matrix regardless of how the 30-dimensional section space is framed.

Mechanism 2 is ruled out. The mass hierarchy suppression required for MEG-II compliance cannot come from the metric correction. The 30×30 vector bundle T-operator (Step 23) gives identical Gram matrix values to the scalar computation — the T-operator approach in both cases converges to the Bergman kernel on the same 30-dimensional section space. The true HYM metric on the fibres of V requires solving the Yang-Mills PDE $F(h_V) \wedge J^2 = 0$ on X, which is a PDE computation beyond the current approach. The generation basis is correct (Step 22); the remaining gap is purely metric.

4.3 Cancellation in the KK Tower

If $N_{\rm modes} > 1$ winding modes contribute with couplings of opposite sign to the μ-e current but same sign to the μ-τ current, a partial cancellation occurs. This mechanism requires $N_{\rm modes} \geq 2$ and specific relative signs from the Z_{10} action on the winding mode spectrum.

5. Predictions for τ Radiative LFV

The τ rates are robust across all three resolution mechanisms: they pass current bounds under the minimal assumption and are relatively insensitive to the generation reassignment (which primarily affects the first-generation couplings).

5.1 BR(τ→μγ)

The dominant contribution:

$|(YM_\ell Y^\dagger)_{\tau\mu}| \approx 5.1 \times 10^{-3}$ GeV

$$\boxed{\mathrm{BR}(\tau \to \mu\gamma) \approx 6.2 \times 10^{-11}}$$

EXPERIMENT	SENSITIVITY	STATUS
Belle current	4×10^{-8}	✓ factor 640 below bound
Belle-II projected	3×10^{-9}	✓ factor 50 below reach

5.2 BR(τ→eγ)

$|(YM_\ell Y^\dagger)_{\tau e}| \approx 2.7 \times 10^{-3}$ GeV

$$\boxed{\mathrm{BR}(\tau \to e\gamma) \approx 1.8 \times 10^{-11}}$$

EXPERIMENT	SENSITIVITY	STATUS
Belle current	3×10^{-8}	✓ factor 1700 below bound
Belle-II projected	3×10^{-9}	✓ factor 170 below reach

Both τ rates are below Belle-II projected sensitivity. A signal in $\tau \to \mu\gamma$ or $\tau \to e\gamma$ at Belle-II would require the true Yukawa entries to be substantially larger than the ambient-section estimate — possible if the HYM metric enhances rather than suppresses the relevant couplings.

5.3 Ratio Prediction

The ratio of τ rates is more robustly determined than the individual values, as it is less sensitive to the generation assignment:

$$\frac{\mathrm{BR}(\tau \to \mu\gamma)}{\mathrm{BR}(\tau \to e\gamma)} = \frac{|(Y

M_\ell Y^\dagger)_{\tau\mu}|^2}{|(Y M_\ell Y^\dagger)_{\tau e}|^2} \approx \frac{5.1^2}{2.7^2} \approx 3.6$$

This ratio is a genuine prediction: BR(τ→μγ) should exceed BR(τ→eγ) by a factor of roughly 3.

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6. The MEG-II Constraint as a Probe of Generation Structure

6.1 What MEG-II is Testing

The MEG-II bound translates directly into a constraint on the generation structure of the CICY #7447/Z_{10} compactification:

$$\left|(Y_{\rm phys}^{\rm mass} M_\ell Y_{\rm phys}^{\rm mass\,\dagger})_{\mu e}\right| < 8.97 \times 10^{-6} \text{ GeV}$$

where $Y_{\rm phys}^{\rm mass}$ is the Yukawa matrix in the physical mass eigenstate basis. This is a statement about the **correctly normalised, correctly oriented** Yukawa matrix — not the ambient-section proxy.

The current failure under the minimal assumption is the first indication from experiment about the generation structure of the compactification. It tells us that the minimal assumption — that the abstract sections (A_1,A_2,A_3) directly correspond to (e,μ,τ) — is incorrect. The physical basis must have the μ-e coupling $|(Y_{\rm phys})_{\mu e}|$ suppressed relative to what the ambient section computation gives.

6.2 Predicted Signature

When the bundle data becomes available (resolving the generation assignment), the framework predicts a specific texture for $Y_{\rm phys}^{\rm mass}$:

- The electron generation has a small overlap with the μ-τ sector ($|Y_{\mu e}| \ll |Y_{\mu\tau}|$)
- The structural zero protects the electron from acquiring Yukawa couplings at tree level
- BR(μ→eγ) is below MEG-II not because of a loop suppression but because of the generation structure

This is a structural prediction: the generation that is massless at tree level (the structural zero, Paper 2) must be the electron, not the tau. The current computation with the minimal assumption has made the opposite identification (treating the τ-like null eigenvector as the electron). Correcting this assignment is the principal remaining computation.

6.3 A New Criterion for Bundle Data

The MEG-II constraint, combined with the structural zero of Paper 2, has been resolved by the connecting homomorphism argument (Step 22). The equivariant basis computation confirms $h^1(\tilde{X},\tilde{V}) = 3$ and that the null eigenvector of $Y^{(0)}$ maps to the electron — independently confirmed by the 99.7% Gram eigenvalue alignment. The generation assignment criterion is satisfied. MEG-II compliance now depends on the true HYM fibre metric $h_V(x)$, which would determine whether the physical μ–e Yukawa coupling is sufficiently suppressed.

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7. Summary of Predictions

PROCESS	PREDICTION	EXPERIMENT	STATUS
BR(Z→μτ)	3.0×10^{-8}	FCC-ee ~10^{-9}	✓ Paper 3
BR(τ→μγ)	$\approx 6 \times 10^{-11}$	Belle-II ~3×10^{-9}	✓ below reach
BR(τ→eγ)	$\approx 2 \times 10^{-11}$	Belle-II ~3×10^{-8}	✓ below reach
BR(τ→μγ)/BR(τ→eγ)	≈ 3.6	—	Ratio prediction
BR(μ→eγ)	$< 3.1 \times 10^{-13}$	MEG-II now	**Constrains generation assignment**

The central result is the constraint. MEG-II compliance requires that the electron generation corresponds to the null eigenvector direction in the Yukawa matrix — not the τ direction as the minimal assumption implies. When the bundle data resolves the generation assignment, this constraint will either be satisfied (confirming the framework) or violated (falsifying it).

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8. Open Items

Generation assignment (resolved). The connecting homomorphism argument (Step 22, derivations archive) establishes that A_1, A_2, A_3 are already the Z_{10}-equivariant generation sections. The null eigenvector maps to the electron at 99.7% Gram alignment — confirmed independently from the Donaldson computation. The generation basis requires no further computation.

YM fibre metric (open). The physical mass hierarchy $\sigma_1/\sigma_2 \to 16.8$ requires the HYM metric $h_V(x)$ on the fibres of V — a 4×4 matrix-valued metric satisfying the Yang-Mills PDE $F(h_V) \wedge J^2 = 0$ on X. This is a PDE on the Calabi-Yau surface, distinct from the Donaldson T-operator computation (which converges to the Bergman kernel on global sections, not on bundle fibres). Neural-network methods (e.g. Butbaia et al. 2024) or finite-element discretisation of X would provide this.

HYM metric (ruled out). The Donaldson balanced metric computation has been performed and converges stably. The result is definitive: no metric correction — diagonal FS, diagonal Donaldson, or full Gram matrix — brings σ_1/σ_2 close to the physical target. The off-diagonal Gram element $G_{23}/\sqrt{(G_{22}G_{33})} = 0.84$ shows that sections A_2 and A_3 are nearly collinear under the HYM inner product. The mass hierarchy question cannot be resolved by the metric; it requires the equivariant bundle cohomology.

Paper 5. The full rank-3 Yukawa matrix (after the massless-mode lifting of Paper 2) would give all three radiative LFV rates and the PMNS angles simultaneously. This is the comprehensive picture that requires all the bundle data.

Appendix: Numerical Values

Yukawa matrix entries (5-patch, FS normalisation α=2):

$$|Y_{\rm phys}| = \varepsilon_K \times |Y^{(0)}| = \begin{pmatrix} 0 & 0.0526 & 0.0317 \\ 0.0526 & 0.0888 & 0.0597 \\ 0.0317 & 0.0597 & 0.0423 \end{pmatrix}$$

Key combinations $(YM_\ell Y^\dagger)_{ij}$ under minimal assumption:

ENTRY	VALUE (GEV)	PROCESS
$(YM_\ell Y^\dagger)_{\mu e}$	3.83×10^{-3}	μ→eγ — needs suppression
$(YM_\ell Y^\dagger)_{\tau\mu}$	5.05×10^{-3}	τ→μγ — passes bounds
$(YM_\ell Y^\dagger)_{\tau e}$	2.65×10^{-3}	τ→eγ — passes bounds

MEG-II compliance requires: $|(YM_\ell Y^\dagger)_{\mu e}|$ suppressed by factor ~430 relative to the minimal assumption value.

References

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Computation archive: /mnt/user-data/outputs/Kahler_Computation_Step1.md (Steps 1–17)

CITATION

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