Topological Closure and the Two-Clock Quantum Embedding

Bell Compatibility and Terminal Record Neutrality within the Selective Transient Field Framework

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Abstract

High-redshift Cosmic Bell experiments severely restrict local-realist accounts in which measurement settings and hidden variables are coordinated by a common cause in their accessible past. They do not test every time-symmetric or all-at-once formulation, and they neither verify nor falsify the Selective Transient Field (STF) framework. Their relevance to STF is architectural: any extension of the framework to quantum measurement must reproduce Bell nonfactorizability while preserving operational no-signaling and measurement independence.

The Clock-Separation Theorem establishes that the oscillating STF amplitude cannot also generate a continuous universal time direction through its normalized gradient. The present framework therefore assigns distinct roles to the scalar's cyclic Compton phase, the record clocks of local systems, and the universal ordering selected by global closure geometry. Following *The Universal Clock Carrier*, the preferred carrier of that ordering is not a new propagating khronon but a boundary-selected constant-mean-curvature (CMC) normal

$$N^\mu_{\rm CMC} = N^\mu_{\rm CMC}[g; \Sigma_{\rm closure}, \Sigma_{\rm terminal}, V_4].$$

Within this architecture we construct a time-symmetric transaction functional. Conditional on an anti-linear outcome-fiber adjoint, a positive matter-state inner product, normalized amplitudes, and ordinary record decoherence, the functional is Hermitian and positive semidefinite and reproduces Born-diagonal weights. Supplying the standard $SU(2)$ singlet state and local quantum instruments then reproduces the Bell joint distribution, no-signaling marginals, and the Tsirelson bound. This is a faithful STF embedding of quantum probability, not yet an STF derivation of it: the topology fixes the $4\pi^2$ normalization of closure but does not by itself determine the amplitudes, their adjoint, or their probabilistic interpretation.

A future heat-death macrocondition can preserve ordinary statistics if it is effectively neutral on finite local record algebras. We state the precise conditional neutrality lemma and its superselection and quantum-field-theory qualifications. The more speculative identification of holographic state capacity with an STF transaction budget, and of failed closure with a gravitating residue, is developed separately in *Holographic Closure Capacity and Transactional Exhaust* so that it does not alter the proof status of the quantum embedding established here.

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1. What the Cosmic Bell experiments constrain

Bell-inequality violations exclude the conjunction of assumptions used to derive Bell factorization. Cosmic Bell experiments strengthened the freedom-of-choice test by using photons emitted by distant astronomical sources to determine the detector settings. In the quasar experiment, the relevant setting-generating emissions occurred billions of years in the past, forcing any local common-past coordination mechanism into a correspondingly remote region of the experiments' joint past.

The directional qualification is essential. These experiments strongly constrain a local-realist mechanism of the form

$$\rho(\lambda \mid a, b) \neq \rho(\lambda)$$

when the dependence is attributed to a common cause propagating forward from the accessible past. They do not directly test an all-at-once boundary-value description, nor do they establish such a description. They therefore leave STF with an obligation rather than a confirmation: the framework must reproduce the observed Bell probabilities without introducing controllable backward signaling or a hidden common-past conspiracy.

The quantum embedding developed below adopts the cleaner of two logically distinct routes. It introduces no Bell hidden variable λ, retains measurement independence, and places the failure of Bell factorization in the nonseparable global quantum state. The STF retarded/advanced geometry supplies a possible home for the doubled probability structure; it is not used as a setting-dependent signal sent into the past.

Status. The Cosmic Bell experiments constrain common-past local-realist coordination. They do not confirm STF. The construction below must stand on its own mathematical consistency.

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2. Clock separation and the present universal carrier

2.1 The gradient-clock obstruction

Let a scalar q with timelike gradient define

$$n_q^\mu = -\frac{\nabla^\mu q}{\sqrt{-\nabla q \cdot \nabla q}}.$$

Along an integral curve of n_q^μ,

$$n_q^\mu \nabla_\mu q = \sqrt{-\nabla q \cdot \nabla q} > 0,$$

up to the overall orientation convention. The generating scalar is therefore strictly monotonic wherever its normalized gradient exists.

The STF amplitude instead oscillates,

$$\phi \simeq A \cos \Theta_I, \qquad \Theta_I \simeq m_s T_U + \delta \pmod{2\pi}.$$

At each turning point $d\phi/dT_U = 0$, the normalized amplitude gradient is undefined and changes chart orientation. The amplitude cannot be the global clock carrier. This is the proved content of clock separation.

The theorem does not uniquely require a second propagating scalar. It requires the carrier of universal ordering to be mathematically distinct from the oscillating amplitude. The current STF carrier program favors a geometric, boundary-selected construction:

$$N^\mu = N^\mu_{\text{CMC}}[g; \Sigma_{\text{closure}}, \Sigma_{\text{terminal}}, V_4].$$

The CMC lapse is fixed elliptically by the geometry and boundary data. It introduces no independently specifiable local clock wave. Accumulated four-volume orders its leaves, but the numerical leaf label is not itself an observable clock rate.

2.2 Three temporal roles that must not be conflated

The phrase "internal clock" has been used for two distinct local structures. They must be notationally separated:

1. $\Theta_I \in S^1$ is the cyclic Compton phase of the oscillating STF field. It records phase modulo one winding.
2. τ_R is the record-ordering parameter of a detector, organism, or other locally closed subsystem. It orders the formation of stable local records and need not equal Θ_I/m_s.
3. T_U, or equivalently the ordering of the selected CMC leaves, is the universal causal order. It is not erased when an internal phase completes a cycle.

These are three roles at two descriptive levels: one global ordering and a family of local cyclic or record clocks. The Clock-Separation Theorem proves that ϕ cannot collapse all three roles into one amplitude. The CMC construction is the present candidate for the global role; its global existence, uniqueness, and full causal completion remain open.

3. A conditional universal transaction functional

3.1 Topological normalization

For a closed loop γ in the sphere of null directions, let T^2_γ be its Hopf preimage. The retarded Hopf and advanced anti-Hopf sectors carry primitive one-forms ω_R and ω_A satisfying

$$\int_{T^2_\gamma} \omega_R \wedge \omega_A = 4\pi^2.$$

This fixes the normalization of a completed topological transaction. It does not determine a matter amplitude or a probability measure.

For a possible outcome k, retain the matter-and-apparatus coefficient $\mathcal{A}_k$:

$$\Psi_k^R = \mathcal{A}_k \omega_R.$$

The required quantum embedding assumes an anti-linear outcome-fiber map

$$J(\mathcal{A}_k \omega_R) = \overline{\mathcal{A}_k} \omega_A.$$

Anti-linearity is physically motivated by the retarded/advanced adjoint relation, but it is not yet derived from the topology of the complexified null cone. This is the open Outcome-Fiber Adjoint problem.

Conditional on J, define

$$D_U(k, \ell) = \frac{1}{4\pi^2} \int_{T^2_\gamma} (\mathcal{A}_k \omega_R) \wedge (\overline{\mathcal{A}_\ell} \omega_A) = \mathcal{A}_k \overline{\mathcal{A}_\ell}.$$

The $4\pi^2$ factor cancels. Its job is to normalize the closure integral; the complex conjugation producing the quadratic diagonal entered through J.

3.2 Coupling closure to records

Attach record vectors $|R_k(\tau_R)\rangle$ to the alternatives:

$$D_{\mathrm{STF}}(k, \ell) = \mathcal{A}_k \overline{\mathcal{A}_\ell} \langle R_\ell(\tau_R) | R_k(\tau_R) \rangle.$$

The record-overlap matrix is a Gram matrix and is positive semidefinite. The full kernel is its Schur (entrywise) product with the rank-one positive semidefinite matrix $\mathcal{A}_k \overline{\mathcal{A}_\ell}$, and is therefore positive semidefinite by the Schur product theorem, under the stated Hilbert-space assumptions. Before stable recording, off-diagonal overlaps permit interference. After environmental decoherence,

$$\langle R_\ell | R_k \rangle \simeq \delta_{k\ell},$$

and the diagonal becomes

$$D_{\mathrm{STF}}(k,k) = |\mathcal{A}_k|^2.$$

For normalized exhaustive alternatives, the imported quantum probability rule then gives

$$p_k = \frac{D_{\mathrm{STF}}(k,k)}{\sum_j D_{\mathrm{STF}}(j,j)} = |\mathcal{A}_k|^2.$$

This is a natural and internally consistent embedding of Born weights. It is not yet a derivation of the Born rule. Such a derivation would additionally have to establish from STF geometry:

- the anti-linear adjoint J on complete outcome fibers;
- the physical positive inner product and amplitude space;
- amplitude normalization and composition;
- additivity for exclusive alternatives;
- the instrument structure of local measurements; and
- the operational or noncontextual probability interpretation of the diagonal.

Status. Hermiticity and positivity are conditional algebraic results. Born compatibility is established. Born derivation remains open.

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4. Bell compatibility and the measured/observed distinction

4.1 Singlet insertion

For two spin-$1/2$ systems, rotational invariance selects the antisymmetric singlet tensor $\epsilon_{\alpha\beta}$. Supplying the normalized singlet state and the usual local projective instruments yields

$$P(A,B \mid \mathbf{a},\mathbf{b}) = \frac{1}{4}(1 - AB\,\mathbf{a}\cdot\mathbf{b}), \qquad A,B \in \{-1,+1\}.$$

Consequently,

$$E(\mathbf{a},\mathbf{b}) = -\mathbf{a}\cdot\mathbf{b}.$$

For linear photon-polarization analyzers with physical angles a and b, the Bloch-sphere angle is doubled and the corresponding expression is

$$E(a,b) = -\cos 2(a-b).$$

The singlet and detector operators are matter-sector quantum data. They are not generated by the $4\pi^2$ closure integral.

4.2 No-signaling

Before Alice receives any record from Bob, her internal record algebra traces over Bob's alternatives. Completeness of Bob's local instrument gives

$$\sum_B \Pi_B^b = I_B,$$

and therefore

$$P(A \mid \mathbf{a}, \mathbf{b}) = \sum_B P(A, B \mid \mathbf{a}, \mathbf{b}) = \frac{1}{2}.$$

The remote setting disappears from the local marginal. This establishes no-signaling within the embedded quantum construction. It is inherited from the nonseparable state together with complete, trace-preserving local instruments; the one-bit topological lemma alone would not suffice, because one controllable bit is still a signal.

4.3 Tsirelson and claim status

For local dichotomic quantum observables, the usual CHSH operator obeys

$$\|\mathcal{B}_{\mathrm{CHSH}}\| \leq 2\sqrt{2}.$$

With the singlet and optimal analyzer settings, the embedded functional reaches that value. STF therefore reproduces the Tsirelson bound once the standard quantum operator algebra is supplied. It does not presently derive the bound from topological closure alone.

4.4 Measured and observed

The two-clock architecture nevertheless contributes a useful interpretive separation:

- **Measured internally:** each detector instantiates a definite, locally ordered record. No controllable influence propagates from the remote setting into its marginal distribution.
- **Observed jointly:** the nonfactorizable correlation becomes accessible when the two records are brought into a shared causal future and compared.
- **Represented universally:** the complete boundary-conditioned history contains the relation between both records without requiring a physical collapse signal to travel between the wings.

This is a reconciliation of local records with global correlations. It is not an independent dynamical selection rule for which outcome occurs.

Status. Bell and Tsirelson compatibility are established conditionally on standard quantum matter and instrument structure. No-signaling is preserved. Outcome actualization remains a separate ontological question.

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5. Terminal record neutrality

5.1 The final-boundary trilemma

For class operators C_α, initial state ρ_i, and terminal effect E_f, a time-neutral decoherence functional takes the form

$$D_f(\alpha, \beta) = \frac{\mathrm{Tr}\left(E_f C_\alpha \rho_i C_\beta^\dagger\right)}{\mathrm{Tr}(E_f \rho_i)}.$$

Three terminal choices must be distinguished:

1. An effectively neutral terminal effect preserves ordinary statistics but selects no particular outcome.
2. A pure record-specific final microstate generally produces ABL-type postselection and changes the probabilities.
3. A high-entropy heat-death macrocondition can preserve ordinary statistics if its action is approximately neutral across the ancient local record alternatives.

The future boundary must therefore be a macrocondition rather than an outcome-specific pure microstate if ordinary Born statistics are to survive.

5.2 Conditional Terminal Record-Neutrality Lemma

Let the regulated Hilbert space factor locally as

$$\mathcal{H} = \mathcal{H}_{\mathrm{cosmic}} \otimes \mathcal{H}_R.$$

If

$$[E_f, I \otimes U_R] = 0$$

for every unitary U_R generating the full irreducible record algebra $\mathcal{B}(\mathcal{H}_R)$, then the commutant theorem implies

$$E_f = E_{\mathrm{cosmic}} \otimes I_R.$$

This is a correct conditional lemma. Heat death does not by itself prove its hypothesis.

If only charge-preserving transformations are permitted, the record representation generally decomposes into superselection sectors and the conclusion weakens to

$$E_f = \bigoplus_q E_q \otimes I_q.$$

Different sectors may carry different terminal weights. Even the factorized form is not sufficient if the rest-sector effect becomes branch-dependent through intervening dynamics. Born recovery therefore requires effective branch neutrality, not merely a formal tensor factorization.

In an infinite-dimensional quantum field theory, I_R is not a trace-class density operator. A heat-death boundary must instead be represented through a regulated macro-subspace,

$$E_f \propto P_{\mathrm{HD}},$$

or through an appropriate algebraic or KMS-type macrostate.

5.3 What neutrality does and does not do

Terminal neutrality allows a future macroboundary to anchor the global boundary-value description without micromanaging laboratory records. It does not select one record. The statement that one closed history is actual is the Single-Transaction Ontology: a framework commitment, not a consequence of the $4\pi^2$ winding number.

The “garbage collection” language may be retained as a metaphor: heat death destroys the practical accessibility of microscopic record pointers through thermalization and scrambling. It has not been shown that a literal memory register is erased, that entropy capacity is thereby recovered, or that only a topological checksum remains as a physical degree of freedom.

Status. Record neutrality is a sufficient conditional structure, motivated but not derived by heat death. Actualization and repeated-trial typicality remain open.

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6. Consolidated claim ledger

Statement	Status
An oscillating amplitude cannot be a global normalized-gradient clock	proved
STF therefore needs a universal carrier distinct from ϕ	conditional entailment within STF
The carrier must be a propagating khronon	false; not required
Boundary-selected CMC ordering is the preferred carrier candidate	derived candidate; global completion open
The Hopf/anti-Hopf pairing has normalization $4\pi^2$	established within the STF topological construction
The conditional transaction kernel is Hermitian and positive semidefinite	algebraic lemma
$4\pi^2$ alone derives the Born rule	false
STF embeds Born weights after an anti-linear adjoint and quantum record structure are supplied	established conditionally
The singlet insertion reproduces Bell probabilities and no-signaling	established conditionally
STF topology independently derives the Tsirelson bound	not established
A record-neutral terminal effect preserves ordinary statistics	conditional lemma
Heat death proves exact record neutrality	not established
Topological closure selects one actual outcome	false on the present formalism

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7. The theorem program exposed by the construction

The paper reduces the remaining work to identifiable problems.

1. **Outcome-Fiber Adjoint Theorem.** Derive, rather than assume, that the Lorentzian real structure mapping the retarded propagator to the advanced propagator maps every complete matter-and-record amplitude to its Hermitian adjoint.
2. **Universal-to-Local Gluing Theorem.** Establish a map from the Hopf-torus closure pairing to a normalized local decoherence or process functional, including composition and instrument normalization.
3. **CMC Carrier Completion.** Prove existence and uniqueness on the relevant STF solution class and incorporate the nonlocal metric response and causal memory kernel consistently.
4. **Terminal Neutrality Dynamics.** Show that the heat-death macrocondition is effectively branch-neutral on finite ancient record algebras.
5. **Repeated-Transaction Typicality.** Explain why one actual global history exhibits Born frequencies across repeated approximately independent trials.

The closure-coding and dark-exhaust programs are downstream conjectures treated in the separate companion paper. They cannot repair a missing upstream gluing or carrier theorem.

7-bis. What would falsify this, and what could not

An exact embedding of the known quantum probability structure is not independently falsifiable by quantum experiment: wherever quantum mechanics is confirmed, the embedding is confirmed with it. That is a property of the claim, not a weakness, and the honest falsification statement has three parts.

Inherited falsifiers. This paper rests on the same physics as *The Universal Embedding Principle*, and inherits that paper's §9 falsifiers rather than adding new ones: (i) curvature- or scalar-background-dependent Bell correlations — in particular, a CHSH deficit tracking the 3.32-year internal phase would kill the common-mode result on which the embedding relies; and (ii) Bell violations modulated by local χ_{local} activation. Either observation would falsify the underlying STF quantum sector, and this construction with it.

A mathematical falsifier. The entire construction is conditional on the anti-linear outcome-fiber adjoint J (§3.1). If the Outcome-Fiber Adjoint problem returns a no-go — a proof that no such J exists on the complexified null cone — the embedding fails by theorem rather than by experiment. Few physical proposals carry that exposure; it is a point in this one's favour.

A neutrality signature. The Terminal Record-Neutrality Lemma (§5.2) has an observational shadow the conditional statement does not by itself supply: if the terminal macrocondition is *not* record-neutral, Born weights would be distorted in a way plausibly dependent on cosmic epoch, and high-redshift quantum measurements — Cosmic Bell tests among them — are where such a distortion would show. Observed Born statistics at high redshift are therefore consistent with, though they do not prove, effective terminal neutrality.

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8. Conclusion

The two-clock STF architecture provides a coherent geometric setting in which local records and global Bell correlations can coexist without a physical superluminal collapse signal. Clock separation prevents the oscillating STF amplitude from being misused as the universal clock, while boundary-selected CMC geometry supplies the present nonpropagating candidate for universal ordering. The

Hopf/anti-Hopf pairing then gives a natural normalized arena for a retarded/advanced transaction functional.

What has been achieved is an embedding with sharp mathematical content. Given an anti-linear outcome-fiber adjoint and standard quantum matter and record structure, the functional is Hermitian, positive, interference-preserving before recording, Born-diagonal after decoherence, Bell-compatible, and no-signaling. The $4\pi^2$ topology normalizes closure; it does not yet choose the quantum adjoint, determine amplitude ratios, or select one actual outcome.

The terminal heat-death boundary can be made compatible with ordinary statistics if it is a record-neutral macrocondition, but neutrality is a condition to derive rather than a fact supplied by the name "heat death."

In metaphorical terms, STF may ultimately describe part of the operating architecture of spacetime. In present mathematical terms, its strongest result is more precise: the two-clock closure geometry can host the known quantum probability structure without contradiction, while exposing exactly which gluing, carrier, and boundary theorems remain to be proved.

References

1. J. S. Bell, "On the Einstein Podolsky Rosen Paradox," *Physics* **1** (1964), 195–200.
2. J. Handsteiner et al., "Cosmic Bell Test: Measurement Settings from Milky Way Stars," *Physical Review Letters* **118** (2017), 060401.
3. D. Rauch et al., "Cosmic Bell Test Using Random Measurement Settings from High-Redshift Quasars," *Physical Review Letters* **121** (2018), 080403.
4. M. Gell-Mann and J. B. Hartle, "Quantum Mechanics in the Light of Quantum Cosmology," in *Complexity, Entropy, and the Physics of Information* (1990).
5. Z. Paz, *Topological Closure on the Complexified Null Cone*, V6.3.
6. Z. Paz, *The Clock-Separation Theorem*, V1.1.
7. Z. Paz, *The Universal Embedding Principle*, V1.1.
8. Z. Paz, *The Universal Clock Carrier*, V1.0.
9. Z. Paz, *One Bit of Destiny — The Theorem of the Unknown*, V1.0.
10. Z. Paz, *Holographic Closure Capacity and Transactional Exhaust*, V1.0.

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