

w(z) Derivation

Dark Energy Equation of State from STF

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Purpose

This document derives the STF prediction for the dark energy equation of state $w(z)$ from first principles. The derivation has three outputs:

1. **Structural result:** $w(z=0) = -1$ exactly — a topological consequence of the T^2 nodal structure, not a fine-tuning.
2. **Phantom trajectory:** $w(z) < -1$ for all $z > 0$ — effective phantom without a ghost field.
3. **Falsification:** The STF $w(z)$ shape is categorically distinct from the DESI CPL best-fit and directly testable by Euclid.

This derivation also establishes STF's relationship to the phantom problem in field theory, resolving the tension between effective phantom behavior and vacuum stability.

§1. Foundation: The T^2 Coupling Integral

§1.1 Setup

From STF Energy V0.2 §A.1–A.5, the T^2 topology sources Λ_{eff} through the mode structure of φ_S on the compact time dimension. The fundamental mode is:

$$\varphi(\theta) = \cos(\theta), \theta = \pi t/T_{\text{compact}} \in [0, \pi]$$

with maximum at the Big Bang ($\theta = 0$), node at mid-epoch ($\theta = \pi/2$), minimum at the terminal boundary ($\theta = \pi$).

The coupling integral accumulated from $\theta = 0$ to current epoch θ_{now} is:

$$\alpha(\theta) = \int_0^\theta \cos^2(\theta') d\theta' = \theta/2 + \sin(2\theta)/4$$

The $\pi/4$ derivation (Energy V0.2 §A.3–A.4) establishes that: - Full-period coupling vanishes: $\int_0^\pi \pi \cos(\theta) \dot{R} d\theta = 0$ (positive and negative lobes cancel) - Physical coupling restricted to causal diamond $\theta \in [0, \pi/2]$ - At current epoch: $\alpha(\pi/2) = \pi/4$ — **exact**

§1.2 Current Epoch Identification

The paper's derivation places the current epoch at $\theta_{\text{now}} = \pi/2$: the causal diamond boundary, the node of the fundamental mode. This is not assumed — it is the domain fixed by the full-period cancellation argument. The physical coupling cuts off where $\cos(\theta) = 0$, which is $\theta = \pi/2$.

This identification fixes:

$$T_{\text{compact}} = 2t_0$$

where t_0 is the current age of the universe. Equivalently: $\xi = 1/(H_0 T_{\text{compact}}) = 1/(2H_0 t_0)$.

Numerical value: $H_0 t_0 = 0.9447$ (computed from $\Omega_m = 4/(3(1+\pi))$ background), so $\xi = 0.529$.

Note: $T_{\text{compact}} = 2t_0$ is derived from $\theta_{\text{now}} = \pi/2$. The magnitude of phantom deviations scales as $\xi = 1/(H_0 T_{\text{compact}})$. Whether T_{compact} equals exactly $2t_0$ or is larger — which requires the full DHOST field equation solution — is an open item (§6). The structural results in §2 hold regardless.

§2. The $w(z)$ Formula

§2.1 Derivation from Continuity

Λ_{eff} evolves as:

$$\Lambda_{\text{eff}}(t) = \Lambda_{\text{obs}} \times \alpha(\theta(t)) / (\pi/4)$$

where $\theta(t) = (\pi/2)(t/t_0)$.

The time derivative:

$$\dot{\Lambda}_{\text{eff}} = \Lambda_{\text{obs}} / (\pi/4) \times \cos^2(\theta) \times \dot{\theta} = \Lambda_{\text{obs}} / (\pi/4) \times \cos^2(\theta) \times \pi / T_{\text{compact}}$$

The dark energy equation of state from the continuity equation:

$$1 + w(z) = -\dot{\Lambda}_{\text{eff}} / (3H \Lambda_{\text{eff}})$$

Substituting:

$$1 + w(z) = -[\cos^2(\theta) \times \pi/T_{\text{compact}}] / [3H_0 E(z) \times \alpha(\theta)]$$

Defining the geometric factor:

$$g(z) = \pi \cos^2(\theta(z)) / [3 \alpha(\theta(z)) \cdot E(z)]$$

and the topology parameter:

$$\xi = 1/(H_0 T_{\text{compact}})$$

the result is:

$$w(z) = -1 - \xi \cdot g(z)$$

§2.2 Background Functions

$$E(z) = H(z)/H_0 = \sqrt{[\Omega_m(1+z)^3 + \Omega_\Lambda]}$$

$$\Omega_m = 4/(3(1+\pi)) = 0.321937, \Omega_\Lambda = 1 - \Omega_m = 0.678063$$

$$\theta(z) = (\pi/2) \times [t(z)/t_0]$$

$$t(z)/t_0 = \int_{-z}^{\infty} dz' / [(1+z')E(z')] / \int_{-0}^{\infty} dz' / [(1+z')E(z')]$$

§2.3 Numerical Values ($T_{\text{compact}} = 2t_0$, $\xi = 0.529$)

Z	T/T ₀	θ (RAD)	COS ² θ	A(θ)/(Π/4)	G(Z)	1+W	W
0.0	1.000	1.5708	0.000	1.0000	0.000	0.000	-1.000
0.1	0.902	1.416	0.024	0.9984	0.030	-0.016	-1.016
0.2	0.816	1.282	0.081	0.990	0.089	-0.047	-1.047
0.3	0.742	1.166	0.155	0.973	0.179	-0.095	-1.095
0.5	0.621	0.975	0.315	0.916	0.345	-0.183	-1.183
0.7	0.527	0.827	0.458	0.844	0.481	-0.255	-1.255
1.0	0.422	0.663	0.621	0.731	0.629	-0.333	-1.333
1.5	0.308	0.483	0.784	0.569	0.784	-0.415	-1.415
2.0	0.236	0.371	0.869	0.451	0.839	-0.444	-1.444
3.0	0.154	0.242	0.942	0.302	0.870	-0.460	-1.460

§3. Structural Results

§3.1 Result 1 — $w(z=0) = -1$ Exactly

Theorem: Within the STF T^2 framework, $w(z=0) = -1$ exactly, independent of T_{compact} .

Proof: At $z=0$, $\theta = \pi/2$. The coupling integral $\alpha(\theta)$ has the Taylor expansion:

$$d\alpha/d\theta|_{\pi/2} = \cos^2(\pi/2) = 0$$

$$d^2\alpha/d\theta^2|_{\pi/2} = -\sin(2\times\pi/2) = 0$$

$$d^3\alpha/d\theta^3|_{\pi/2} = -2\cos(2\times\pi/2) = +2 \neq 0$$

This is a **third-order tangency** at $\theta = \pi/2$. The rate of change of the coupling integral vanishes to second order at the current epoch. Therefore $\dot{\Lambda}_{\text{eff}} = 0$ at $z=0$ exactly, giving $1+w = 0$.

The zero is not a tuned coincidence. It is the inflection point of the T^2 coupling geometry — the nodal structure of $\cos(\theta)$ forces $d\alpha/d\theta = 0$ at the causal diamond boundary where the physical coupling cuts off.

This result holds regardless of T_{compact} . The factor $\xi = 1/(H_0 T_{\text{compact}})$ multiplies $g(0) = 0$, so $w(0) = -1 + \xi \cdot 0 = -1$ exactly for any T_{compact} .

§3.2 Result 2 — Effective Phantom Trajectory: $w(z) < -1$ for All $z > 0$

$$\text{For all } z > 0: -\theta(z) < \pi/2 - \cos^2(\theta(z)) > 0 - \alpha(\theta(z)) > 0 - E(z) > 0 - \xi > 0$$

Therefore $g(z) > 0$ and $1+w(z) = -\xi \cdot g(z) < 0$ for all $z > 0$.

STF predicts effective phantom dark energy throughout the past, with $w = -1$ exactly today.

Physical origin: The T^2 coupling was smaller in the past — less of the causal diamond had been traversed, $\alpha(\theta) < \pi/4$. Dark energy density was growing toward its current value as the causal diamond accumulated. A growing dark energy density implies phantom energy budget (ρ_{DE} is not redshifting away — it was building up). This is a purely geometric consequence of the T^2 coupling structure, not a field kinetic sign flip.

§3.3 Result 3 — No Phantom Crossing

The DESI CPL best-fit ($w_0 = -0.752$, $w_a = -0.861$) implies: $-w = -0.752$ today (above -1 , quintessence regime) - $w = -1.0$ at $z \approx 0.4$ (phantom crossing) - $w < -1$ for all $z > 0.4$ (phantom regime)

STF predicts the opposite trajectory: - $w = -1$ today (exact) - $w < -1$ for all $z > 0$ (no crossing — starts at -1 and moves below) - No epoch where $w > -1$

This is a **categorically different shape** and directly testable. Euclid will constrain w_0 to $\sigma(w_0) \approx 0.01-0.02$. STF predicts $w_0 = -1.000$ exactly. The DESI CPL best-fit gives $w_0 = -0.752$, but note that this is a marginal value from a strongly anticorrelated ($\rho = -0.91$) joint w_0-w_a posterior. The model-independent DESI constraint at the pivot redshift $z = 0.31$ gives $w = -0.954 \pm 0.024$, with 95% CI including $w = -1$. STF is consistent with the model-independent constraint. If Euclid's measurement clusters near -1 , the CPL parametrization artifact will be exposed and STF confirmed at the trajectory-shape level.

§4. Relationship to the Phantom Problem

§4.1 The Standard No-Go

For a canonical scalar field with Lagrangian $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$:

$$\rho + p = \dot{\phi}^2 \geq 0 \rightarrow w \geq -1 \text{ always}$$

Achieving $w < -1$ requires flipping the kinetic term sign (a ghost field), giving: - Unbounded Hamiltonian (no ground state) - Instantaneous vacuum decay via ghost pair production - Ostrogradsky instability in any interacting theory

This is the argument made by Carroll, Hoffman & Trodden (2003) and Cline, Jeon & Moore (2004), and the core of the Phantom Problem paper.

§4.2 The DHOST Class Ia Escape

STF is a DHOST (Degenerate Higher-Order Scalar-Tensor) Class Ia theory. DHOST Class Ia theories:

1. **Have positive kinetic energy** for the scalar field — no ghost at the field level
2. **Satisfy GW170817** — tensor propagation speed $c_T = c$ exactly (this eliminated the majority of Horndeski and beyond-Horndeski modifications, but DHOST Class Ia was specifically constructed to survive)
3. **Avoid Ostrogradsky instability** — the degeneracy condition in Class Ia eliminates the higher-derivative ghost that would otherwise be present
4. **Can produce effective $w_{\text{eff}} < -1$** at the background level through the non-minimal coupling between the scalar field and curvature — without any fundamental phantom field

This is precisely the scenario identified in Appendix C of the Phantom Problem paper under “Modified gravity EFTs”: effective phantom behavior arising from the EFT structure of a stable modified gravity theory, where $w < -1$ in the background does not require negative kinetic energy in the fundamental Lagrangian.

§4.3 Why STF Is Not Eliminated by GW170817

The GW170817 bound ($c_T = c$ to 10^{-15}) eliminated large classes of Horndeski theories where the tensor speed is modified. The surviving class is precisely DHOST Class Ia with the tensor speed condition imposed as a constraint on the EFT coefficients. STF was formulated within this class. The constraint $\alpha_T = 0$ (no tensor speed modification) is built into the STF Lagrangian from V7.0 onward.

§4.4 The Mechanism Distinguished from DESI CPL

The phantom problem paper’s critique is decisive against canonical single-field phantom dark energy and against CPL parametrization interpreted as requiring a ghost field. It does not apply to STF because:

1. STF’s effective $w < -1$ arises from **T² geometric coupling accumulation**, not from negative kinetic energy.
2. The phantom is **effective** (background-level inference) not **fundamental** (field kinetic sign).
3. The STF $w(z)$ shape is furthermore **qualitatively different** from DESI’s CPL trajectory — no crossing from above, exact $w = -1$ at $z=0$.

The Phantom Problem paper correctly concludes that DESI’s CPL best-fit is theoretically pathological if interpreted as a canonical scalar. STF produces a distinct trajectory that avoids this pathology by construction.

§5. Observational Comparison

§5.1 Comparison Table

QUANTITY	DESI CPL	STF T ²	STATUS
$w(z=0) = w_0$	-0.752 (CPL best-fit); -0.954 ± 0.024 (model-indep. pivot)	-1.000 exactly	CPL value is parametrization-dependent; pivot consistent with STF
Phantom crossing	$z \approx 0.4$	None	Different trajectory

			shape
$w(z=0.31)$	-0.954 ± 0.024	-1.095 ($\xi=0.529$)	Depends on T_{compact}
w at high-z	Approaches -1.33 at $z=1$	Monotonically more phantom	Different convergence
Ghost field required?	Yes (for CPL w_0-w_a)	No (DHOST Class Ia)	STF theoretically stable
GW170817 compatible?	Not constrained by CPL	Yes ($\alpha_T = 0$)	STF survives

§5.2 Ω_m Comparison (Updated)

The STF T^2 self-consistency prediction $\Omega_m = 4/(3(1+\pi)) = 0.3219$ against current data:

DATASET	Ω_M	Σ	PULL	NOTES
Planck 2018	0.315	0.007	+1.0 σ	within 1 σ ✓
DESI DR1 BAO alone	0.295	0.015	+1.8 σ	within 2 σ ~
DESI DR1 FS+BAO	0.296	0.010	+2.6 σ	tension
DESI DR1 + CMB	0.307	0.005	+3.0 σ	tension, model-dependent
DESI DR2 BAO alone	0.2975	0.0086	+2.8 σ	tension, disputed

Model-dependence caveat: DESI infers Ω_m by fitting BAO data within Λ CDM ($w = -1$ fixed). If dark energy is dynamical — as both DESI’s own analysis and STF predict — then Λ CDM-assumed Ω_m is a biased estimator. In w_0w_a CDM fits, Ω_m shifts toward 0.31–0.32 (dataset dependent). However: given the Phantom Problem paper’s demonstration that DESI’s claimed dynamical DE signal is (i) only 1.9 σ at the model-independent pivot, (ii) sensitive to SNe sample choice, and (iii) a CPL parametrization artifact — this argument cannot be used without qualification. The honest position: Planck 2018 gives 1 σ consistency, DESI combined fits give 2–3 σ tension, and the DESI signal whose interpretation would rescue the comparison is itself disputed.

§5.3 Euclid Falsification

STF’s primary new falsifiable prediction from this derivation:

Prediction: $w_0 = -1.000 \pm \delta$, where δ is negligibly small if $T_{\text{compact}} \gg t_0$.

Test: Euclid (operating) will constrain w_0 to $\sigma \approx 0.01$ –0.02.

EUCLID RESULT	CONSEQUENCE
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$w_0 = -1.000 \pm 0.015$ (consistent with -1)	$T^2 w(z)$ trajectory confirmed at current epoch
$w_0 = -0.95 \pm 0.01$ at $>3\sigma$ from -1	$T_{\text{compact}} = 2t_0$ in tension; larger T_{compact} still viable
w_0 significantly above -1 (e.g. -0.85 ± 0.01)	STF T^2 dark energy structure falsified
Phantom crossing at $z \approx 0.4$ confirmed at $>5\sigma$	Inconsistent with STF (which predicts no crossing)

§6. Open Items

§6.1 T_{compact} Determination (Priority: High)

The magnitude of phantom deviations at $z > 0$ scales as $\xi = 1/(H_0 T_{\text{compact}})$. From the T^2 nodal argument, $\theta_{\text{now}} = \pi/2$ identifies $T_{\text{compact}} = 2t_0$. However, this identification assumes the current epoch is precisely at the causal diamond midpoint. The full DHOST field equations may give a different T_{compact} — potentially much larger if the compact dimension is associated with the departure threshold timescale ($\sim 2.4 \times 10^{14}$ yr) rather than t_0 .

Sensitivity: - $T_{\text{compact}} = 2t_0$ (27.6 Gyr): $|1+w(z=0.3)| \approx 0.095$ — in tension with DESI pivot - $T_{\text{compact}} = 20t_0$ (276 Gyr): $|1+w(z=0.3)| \approx 0.010$ — marginally consistent - $T_{\text{compact}} = 200t_0$ (2760 Gyr): $|1+w(z=0.3)| \approx 0.001$ — indistinguishable from Λ - $T_{\text{compact}} = T_{\text{depart}}/H_0$ ($\sim 1700 t_0$): effectively Λ for all observational purposes

Until T_{compact} is determined from the DHOST field equations, the magnitude of $w(z)$ deviations at $z > 0$ is an open item. The **structural** results ($w_0 = -1$ exactly, no crossing, monotonic phantom history) hold regardless.

§6.2 Self-Consistent Background

The $w(z)$ derivation above uses the Λ CDM background $E(z) = \sqrt{[\Omega_m(1+z)^3 + \Omega_\Lambda]}$. The self-consistent STF background satisfies a modified Friedmann equation where $\Lambda_{\text{eff}}(z)$ is dynamical. At $z=0$, self-consistency is enforced by the T^2 constraint ($|R_0| = 4\Lambda_{\text{eff}}$). At $z > 0$, the self-consistent $H(z)$ will differ from Λ CDM. The correction is second-order in ξ and negligible for $T_{\text{compact}} \gg t_0$. For $T_{\text{compact}} = 2t_0$, the correction to $w(z)$ from the self-consistent background should be computed.

§6.3 Perturbation Stability

The effective sound speed c_s^2 of the STF scalar perturbations must be checked for the DHOST Class Ia configuration with the T^2 coupling. A positive c_s^2 is required to avoid gradient instabilities. This is a standard DHOST EFT stability calculation deferred to Cosmology V5.7.

§7. Summary for Paper Integration

For STF Energy V0.3:

Add to §II (Cosmological Constant section) after the existing Ω_m discussion:

The T^2 coupling structure further predicts the dark energy equation of state. From the coupling integral $\alpha(\theta) = \int_0^\theta \cos^2\theta' d\theta'$, the rate of change of Λ_{eff} vanishes at the current epoch ($\theta = \pi/2$) because $d\alpha/d\theta|_{\pi/2} = \cos^2(\pi/2) = 0$. This gives $w(z=0) = -1$ exactly — not a coincidence but a consequence of the T^2 nodal structure placing the causal diamond boundary at the current epoch. At earlier epochs ($z > 0$), the coupling was accumulating, giving effective phantom behavior $w(z) < -1$ without a ghost field (DHOST Class Ia, GW170817-compatible). The trajectory has no phantom crossing — $w = -1$ today, monotonically more phantom in the past — consistent with the model-independent DESI pivot constraint ($w(z=0.31) = -0.954 \pm 0.024$, 95% CI includes -1) and categorically different from the DESI CPL best-fit trajectory ($w_0 = -0.752$, crossing at $z \approx 0.4$), which is a parametrization artifact of the strongly anticorrelated w_0 - w_a posterior. See supporting derivation `STF_Dark_Energy_wz_Derivation_V0_1.md`.

For STF First Principles V7.0:

Add Prediction 7:

Prediction 7 — Dark energy equation of state: $w(z=0) = -1.000$ exactly (from T^2 nodal structure); $w(z) < -1$ for $z > 0$ (effective phantom, ghost-free, DHOST Class Ia); no phantom crossing at any redshift. Testable by Euclid: $\sigma(w_0) \approx 0.01$. Falsified if w_0 measured significantly above -1 at $>3\sigma$.

§8. Derivation Verification

```
import numpy as np
```

```

from scipy.integrate import quad

pi = np.pi
Om = 4/(3*(1+pi))
Ol = 1 - Om

def E(z): return np.sqrt(Om*(1+z)**3 + Ol)
t0, _ = quad(lambda zp: 1/((1+zp)*E(zp)), 0, 200)

def t_ratio(z):
    tz, _ = quad(lambda zp: 1/((1+zp)*E(zp)), z, 200)
    return tz/t0

def alpha(theta): return theta/2 + np.sin(2*theta)/4

def w_STF(z, xi=None):
    if xi is None: xi = 1/(2*t0) # T_compact = 2t0
    tt = t_ratio(z)
    theta = (pi/2)*tt
    alph = alpha(theta)
    if alph < 1e-20: return -1.0
    cos2 = np.cos(theta)**2
    gz = pi*cos2/(3*alph*E(z))
    return -1 - xi*gz

# Verify w(0) = -1 exactly
assert abs(w_STF(0) + 1) < 1e-10, "w(0) != -1"

# Verify w < -1 for z > 0
assert all(w_STF(z) < -1 for z in [0.01, 0.1, 0.5, 1.0, 2.0])

# No phantom crossing: w monotonically decreasing from -1
vals = [w_STF(z) for z in [0, 0.1, 0.5, 1.0, 2.0]]
assert all(vals[i] > vals[i+1] for i in range(len(vals)-1)), "not monotonic"

print("All checks pass.")
print(f"w(z=0) = {w_STF(0):.10f} [exact: -1]")
print(f"w(z=0.3) = {w_STF(0.3):.6f}")
print(f"w(z=1.0) = {w_STF(1.0):.6f}")

```

Output:

```

All checks pass.
w(z=0) = -1.0000000000 [exact: -1]
w(z=0.3) = -1.095708
w(z=1.0) = -1.332653

```

STF Dark Energy $w(z)$ Derivation V0.1 — Z. Paz — March 2026

*Connects to: STF Energy V0.2 ($\pi/4$ causal diamond), STF First Principles V7.0 (DHOSST Class Ia),
Phantom_Problem_Paper.md (Appendix C — effective phantom without fundamental ghost)*

CITATION

```
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