Dark Matter as Geometry: The Selective Transient Field Framework for a Unified Dark Sector

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Abstract

We present the Selective Transient Field (STF) framework as a candidate unified dark sector theory in which both dark matter and dark energy emerge from a single scalar field — the breathing mode of six compact extra dimensions in a 10D Einstein-Gauss-Bonnet compactification on a Calabi-Yau threefold (CICY #7447) with Z_{10} free quotient structure. The framework has three tiers of achievement. **Forward results (theorem-level):** The Lagrangian is ghost-free on all backgrounds, gravitational wave speed satisfies c_T = c exactly (GW170817-compatible), and a no-go theorem proves that MOND-like dynamics cannot emerge from any local analytic scalar theory — forcing any viable completion into a nonperturbative collective phase. **Scale-setting (geometry-constrained):** The scalar mass m_s = 3.94×10^{-23} eV and coupling $\zeta/\Lambda = 1.35 \times 10^{11}$ m² are determined by the compactification geometry with no parameters fitted to observations; the compactification manifold CICY #7447 is the unique CICY admitting the required free Z_{10} quotient. These scales place perturbative breakdown at astrophysically interesting values (parsec to kiloparsec). **Phenomenological bridge (structural conjectures):** The connection between the microscopic Lagrangian and galactic dynamics requires structural assumptions whose status ranges from "physically motivated" to "uncomputed": a fold-catastrophe mesoscopic effective theory giving the universal X^{3/2} phonon exponent (conditional on a cubic-tangency assumption — Paz 2026e §2.2a), a marginal-stability closure deriving the inverse phonon-baryon coupling γ_DM(M_b) = c³/((ζ/Λ)(GM_b a₀)^{1/4}) under three zero-parameter conditions (the Toomre marginal-stability condition $Q \simeq 1$ is empirically anchored but its ecology-loop closure — whether STF dynamics drive systems toward $Q \simeq 1$ — is open, Paz 2026e §4.2), and a cross-disformal matter coupling that is not generated by the compactification — its UV origin is the primary remaining structural gap. The argument runs forward from the Lagrangian in the first two tiers and relies on structural choices in the third; this paper maps the boundary between them explicitly. On cosmological scales, the oscillation-averaged field mimics pressureless cold dark matter ($\langle w \rangle = 0$, $\rho \propto a^{-3}$); on galactic scales, the nonperturbative collective phase produces MOND-like dynamics. The dark energy equation of state $w(z = 0) = -1$ follows exactly from the T² nodal

structure. **Keystone calculation for the unification claim:** while background-level cosmological equivalence to CDM is established, the perturbation-level demonstration remains the load-bearing calculation. The vacuum scalar mass $m_s = 3.94 \times 10^{-23}$ eV lies in the range where a **bare canonical ultralight scalar** would be strongly constrained by Lyman-α forest data. STF is not canonical ULDM. The canonical ULDM transfer function is the $S_{cos} = 0$ failure limit of STF, not the full theory prediction — it assumes the gravitating perturbation is the bare Schrödinger-Poisson scalar mode, an assumption STF's galactic sector explicitly denies through the field-normalization theorem. STF predicts a different long-distance variable: a dressed condensate-current/geometry mode generated by the self-referential curvature-rate coupling. The keystone calculation is therefore to derive the STF-native cosmological screening factor $S_{cos}(k, z)$ in the renormalized fuzzy-pressure term $k^4/(4m_s^2a^4) \to k^4/[(1 + S_{cos}(k, z)) \cdot 4m_s^2a^4]$, and determine whether $S_{cos}(k, z \simeq 2\text{-}5) \gtrsim 3 \times 10^5$ on Lyman-α scales ($k \sim 0.5\text{-}10\ h\ \mathrm{Mpc}^{-1}$). If $S_{cos} = 0$, STF reduces to canonical ULDM at the vacuum mass and the unified dark-matter claim fails. If S_{cos} is large, Lyman-α constrains the STF collective response, not the bare scalar mass. Until this calculation closes, the dark-sector unification claim should be read as identificatory rather than fully dynamical. The galactic MOND-like sector, controlled by the nonperturbative collective phase and the coupling ζ/Λ, remains structurally separable from this cosmological perturbation test. **Two separable claims.** Throughout this paper we distinguish: **(A) the class-level claim** that the convergent observational evidence — null direct detection entering the neutrino fog, the wide binary anomaly at a_0, the universal MOND Depth Index, and the persistent $a_0 = cH_0/(2\pi)$ connection across galaxy types — has broken the explanatory monopoly of the particle dark matter paradigm: non-particle alternatives (including but not limited to field-theoretic ones) are now as responsible to investigate as continued particle searches. We hold this claim with high confidence; it does not require the STF or any specific framework to be correct. **(B) the specific-mechanism claim** that the STF is the field-theoretic realization of the alternative — partially established with the explicit closure gaps documented in §VI. Argument A justifies the research program; Argument B is the paper's principal scientific commitment, held to the calibrated tier structure above. A reader could accept Argument A and remain agnostic about which (or whether any) specific non-particle mechanism is correct. We hold the two arguments to different epistemic standards: A is methodological (justifies looking elsewhere); B is mechanistic (proposes a specific where).

We conduct a systematic comparison of the STF prediction with seven competing dark matter frameworks — cold dark matter (ΛCDM), Modified Newtonian Dynamics (MOND), fuzzy/ultralight dark matter, warm dark matter (WDM), self-interacting dark matter (SIDM), emergent/entropic gravity, and superfluid dark matter — identifying specific observational failures and theoretical incompleteness in each. We compile and contextualize recent observational developments that collectively strain the WIMP dark matter hypothesis and highlight unexplained galactic regularities: (i) null results from the LZ direct detection experiment after 417 live days with 10 tonnes of liquid xenon (Akerib et al. 2025), now entering the irreducible neutrino fog — noting that this detection floor constrains WIMPs specifically and does not bound ultralight scalars, axions, or sterile neutrinos; (ii) the $>3\sigma$ wide binary gravitational anomaly at accelerations below 10^{-10} m/s^2 from 36 systems with high-precision 3D velocities (Chae et al. 2026), independently confirmed by Hernandez & Kroupa (2025); (iii) the universal MOND Depth Index classification of stellar systems from star clusters to galaxies (Eappen & Kroupa 2026); (iv) growing mainstream interest in scalar field dark matter (Matos & Ureña-López 2025); and (v) evidence from ΛCDM hydrodynamical simulations that the inferred acceleration parameter a_0 increases with redshift (Mayer et al. 2022), consistent with the STF prediction $a_0(z) = cH(z)/(2\pi)$.

The STF galactic inverse-coupling parameter γ_{DM}, initially phenomenological, is derived through a marginal-stability closure: the fold catastrophe of the bounded baryonic response gives the universal $X^{3/2}$ phonon exponent, and three zero-parameter conditions at the MOND radius (Toomre marginal stability $Q \simeq 1$, disformal saturation $\delta\tilde{g}/g \sim 1$, crossover anchoring at r_{a_0}) fix the coefficient, yielding $\gamma_{DM}(M_b) = c^3/((\zeta/\Lambda)(GM_b a_0)^{1/4})$ under one structural assumption. We construct the galactic effective field theory anchored in the STF's perturbative breakdown structure. The derived trigger variable $h(r)$ — which grows as r^3 because $\hat{B} \propto r^9$ through the Weyl tensor profile — diagnoses where the perturbative cross-disformal expansion fails ($h \sim 1$ at ~1 pc, $h \sim 10^{12}$ at 10 kpc). We prove via functional renormalization group analysis that the MOND constitutive law $P(X) \propto X^{3/2}$ cannot arise as an IR fixed point of the bare scalar flow in a shift-symmetric LPA truncation, blocking the simplest naive completion and motivating a collective-phase description. We then construct a baryon-dressed response model for the deep $h \gg 1$ regime. Explicit computation establishes that bare STF cumulants are Gaussian, that standard polytropic baryonic response gives the wrong signs for $\Gamma = 5/3$, and that bounded baryonic compressibility gives the correct sign structure — screened quartic and positive sextic, proven for all barrier exponents $p > 0$. The correct collective variable is the bounded response fraction $y \in [0,1]$, with a crossover driver anchored at the galactic radius where Newtonian acceleration equals a_0. The persistent sextic stabilizer required for the cubic-EOS window is derived from self-gravitational feedback: the Toomre/Jeans stability boundary at $\rho_J = \kappa^2/(2\pi G)$ provides a geometric, non-screening wall whose normalized sextic invariant exceeds 0.1 for all wall exponents $s < 13.4$. With this, the cubic-EOS window opens from ~6 kpc outward, giving $\varepsilon \propto n^3 \rightarrow P(X) \propto X^{3/2} \rightarrow$ the MOND nonlinear Poisson equation, from which the 1/r exterior force law and the baryonic Tully-Fisher relation $v^4 = GMa_0$ follow as mathematical theorems. The derivation chain from STF geometry to the MOND force law is now complete in the London sense for disk-dominated systems — every phenomenological step is identified and characterized — conditional on one broad parameter range (wall exponent $s < 13.4$) and on the applicability of Toomre-type self-gravitational stability. The microscopic bridge from the parent Lagrangian to the collective-phase equation of state (the "Gor'kov" derivation) remains the central open problem; the fold-catastrophe analysis proves that the phonon exponent 3/2 is universal, and the marginal-stability closure (Toomre $Q \simeq 1$ plus disformal saturation

at the MOND radius) derives the inverse phonon-baryon coupling $\gamma_{DM}(M_b) = c^3/((\zeta/\Lambda)(GM_b a_0)^{1/4})$ with zero free parameters, under one structural assumption (disformal response saturates at O(1) in the nonperturbative regime). The complete chain from STF microphysics to the MOND field equation is: perturbative breakdown → collective phase transition → cubic equation of state → nonanalytic phonon action → nonlinear Poisson → flat rotation curves. A temperature-dependent suppression variable accounts for cluster-scale decoherence. A two-parameter phase diagram classifies all astrophysical systems from the solar system to galaxy clusters. Of the framework's testable predictions, one is uniquely STF: the MOND acceleration scale should evolve with redshift as $a_0(z) = cH(z)/(2\pi)$ with proportionality constant $c/(2\pi)$ set by the derived coupling ζ/Λ — conditional on the Level 3 galactic-condensate program closing favorably (V7.9 §I.5 Path A framing); the dimensional-analytic prediction at z=0 is robust (97% match against SPARC), but the explicit redshift dependence depends on whether $\gamma_{eff}(z)$ cancels the H(z) scaling in the closure or whether matching to $\sqrt{\Lambda}$ rather than H is the correct cosmic boundary (open Level 3 question). Two others — soliton core sizes $r_c \sim \hbar/(m_s v)$ and wide binary transition widths — are shared with the broader ultralight scalar class at the corresponding mass, and become STF-distinctive only if the activation dynamics of the Q-phase produce a unique transition morphology.

Keywords: dark matter, scalar field cosmology, MOND acceleration scale, Tully-Fisher relation, galaxy rotation curves, dark energy, unified dark sector, Calabi-Yau compactification, ghost-free scalar-tensor gravity, wide binary anomaly, disformal phase transition, emergent galactic dynamics

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I. Introduction

I.A The Dark Matter Problem

The evidence for gravitational effects beyond visible matter is overwhelming and has been accumulating for nearly a century. Zwicky (1933) first inferred a mass discrepancy in the Coma Cluster from galaxy velocity dispersions. Rubin & Ford (1970) established that spiral galaxy rotation curves remain flat at radii far beyond the visible disk, where Keplerian dynamics predicts a declining velocity profile $v \propto r^{-1/2}$. The cosmic microwave background (CMB) angular power spectrum requires a non-baryonic matter component with $\Omega_{DM} h^2 = 0.120 \pm 0.001$ (Planck Collaboration, Aghanim et al. 2020). Baryon acoustic oscillations (BAO), gravitational lensing of galaxy clusters, and the large-scale matter power spectrum independently corroborate the existence of a gravitationally dominant, optically dark component comprising approximately 27% of the universe's energy budget.

The standard cosmological model (ΛCDM) interprets these observations as evidence for a new species of matter — cold, collisionless, non-baryonic particles that interact with ordinary matter exclusively through gravity. This interpretation has been spectacularly successful on scales larger than ~1 Mpc, where N-body simulations reproduce the observed cosmic web, cluster mass functions, and CMB anisotropies with precision. However, the particle dark matter hypothesis faces three categories of challenge that remain actively debated: persistent null results in direct detection experiments, unexplained regularities in galactic dynamics, and small-scale structure problems that resist resolution through baryonic feedback.

I.B The Detection Crisis

The search for dark matter particles has been dominated by the WIMP (Weakly Interacting Massive Particle) hypothesis, motivated by connections to supersymmetry and the “WIMP miracle” — the observation that a particle with weak-scale mass and interaction strength naturally produces the observed relic density. Experimental efforts have been characterized by a rapid increase in sensitivity spanning many orders of magnitude.

The world’s most sensitive direct detection experiment, LUX-ZEPLIN (LZ), operates a 10-tonne liquid xenon time projection chamber one mile underground at the Sanford Underground Research Facility in South Dakota (LZ Collaboration, Akerib et al. 2025). Based on 4.2 tonne-years of total exposure (280 live days), the experiment placed world-leading constraints on spin-independent WIMP-nucleon cross-sections: $\sigma_{SI} < 2.2 \times 10^{-48}$ cm² at the 90% confidence level for a WIMP mass of 40 GeV/c², with no evidence for an excess over expected backgrounds (LZ Collaboration, PRL 135, 011802, 2025). A subsequent analysis extending to lower masses (3–9 GeV/c²), based on 417 live days of data collected through April 2025, similarly found no WIMP signal (LZ Collaboration, arXiv:2512.08065). This analysis also achieved the first statistically significant detection of boron-8 solar neutrinos via coherent elastic neutrino-nucleus scattering (CEνNS) in a dark matter detector — marking the experimental entry into the “neutrino fog,” an irreducible background of neutrino interactions that mimics dark matter signals and fundamentally limits future WIMP sensitivity at low masses.

Concurrent null results from XENONnT (Aprile et al. 2023) and PandaX-4T (Bo et al. 2025) exclude overlapping parameter space. At the Large Hadron Collider, the conclusion of Run 3 (2022–2025) has pushed exclusion limits for supersymmetric partners beyond 2.4 TeV for gluinos and beyond 1 TeV for charginos and neutralinos across multiple search channels, with no detection (ATLAS and CMS Collaborations 2025). The expected supersymmetric “natural” mass range is substantially excluded.

I.C The Galactic Regularity Problem

While direct detection experiments search for particles, a separate body of evidence poses a deeper conceptual challenge: the observed dynamics of galaxies follow simple, universal scaling relations that the CDM paradigm does not predict and struggles to reproduce.

Milgrom (1983) identified the pivotal observation: galaxy rotation curves begin to deviate from Newtonian expectations at a characteristic acceleration scale $a_0 \approx 1.2 \times 10^{-10}$ m/s². Above this scale, dynamics are Newtonian. Below it, the effective gravitational acceleration is enhanced relative to the Newtonian prediction. This transition is universal across all galaxy types — from gas-rich dwarfs to massive ellipticals, from low-surface-brightness galaxies to high-surface-brightness spirals.

The radial acceleration relation (RAR), established by McGaugh, Lelli & Schombert (2016) using 153 galaxies from the Spitzer Photometry and Accurate Rotation Curves (SPARC) sample, quantifies this regularity: the observed gravitational acceleration g_obs is a one-parameter function of the baryonic Newtonian acceleration g_bar, with a single transition scale a_0. The scatter in this relation is remarkably small — smaller than the scatter in the baryonic Tully-Fisher relation, and smaller than what CDM simulations with baryonic feedback have reproduced. The relation has been extended to elliptical galaxies (Faber-Jackson relation, $L \propto \sigma^4$) and dwarf spheroidals with consistent results.

The numerical coincidence $a_0 \approx cH_0/(2\pi)$, connecting galactic dynamics to the cosmological expansion rate, has been noted by multiple authors (Milgrom 1983, Sanders 1990, Famaey & McGaugh 2012) but has no explanation within ΛCDM. If dark matter is a particle, there is no reason for the particle's gravitational influence to produce a universal transition at an acceleration that happens to equal the speed of light divided by the age of the universe.

I.D The Wide Binary Anomaly

Most recently, the gravitational anomaly at a_0 has been detected in a new environment: wide binary stars. Chae et al. (2026) assembled a sample of 36 wide binaries with high-precision three-dimensional velocities — combining Gaia proper motions with ground-based radial velocity measurements — and performed Bayesian inference on the gravitational parameter $\gamma \equiv G_{eff}/G_N$ for each system. They reported $\gamma \approx 1.6 \pm 0.2$ at internal accelerations below $\sim 10^{-10}$ m/s², constituting a >3σ detection of deviation from Newtonian gravity (Chae et al., arXiv:2601.21728). Independently, Hernandez & Kroupa (2025) found a 22% boost in relative velocities at projected separations above 3000 au using Gaia DR3 data, consistent with MOND predictions.

The wide binary environment is uniquely clean: isolated pairs in the solar neighborhood have negligible dark matter density by any estimate, so CDM predicts strictly Newtonian dynamics. The result remains actively debated and should be considered unsettled. Banik et al. (2024) analyze the same Gaia DR3 data with a different statistical methodology and claim a 19σ preference for Newtonian gravity, modeling kinematic contaminants with six nuisance parameters rather than removing them from the sample. The Chae and Banik analyses differ in sample selection, contamination treatment, and statistical framework, and neither has been independently replicated with the methodology of the other. Hernandez et al. (2024) provide an independent analysis supporting the anomaly. Resolution likely requires Gaia DR4 with improved radial velocities and larger clean samples. The STF prediction is consistent with the Chae/Hernandez results if confirmed.

I.E Scope and Organization of This Paper

In this paper we present the Selective Transient Field as a framework for the dark sector that proposes structural responses to all three categories of challenge: the detection crisis (there is no particle), the galactic regularity problem (a_0 is connected to H_0 through independently constrained parameters), and the wide binary anomaly (the nonperturbative transition activates at exactly a_0). The cosmological sector is developed; the galactic sector is a structured research program with identified but unsolved derivation targets.

Argument structure of this paper. The motivations developed in §I.A–I.D — null direct detection entering the neutrino fog, the wide binary anomaly at a_0, the universal MOND Depth Index, and the persistent $a_0 = cH_0/(2\pi)$ connection across galaxy types — collectively support a **class-level claim**: the convergent evidence has broken the explanatory monopoly of the particle dark matter paradigm. **Non-particle alternatives are now as responsible to investigate as continued particle searches.** This is a methodological claim about where the field's investigative obligation now lies, not an ontological claim that particle dark matter is excluded. **This argument is independent of any specific alternative framework.** A reader could accept the class-level claim and remain agnostic about which (or whether any) specific non-particle mechanism — STF, fuzzy DM, axion-like, modified gravity with a scalar mode, emergent inertia, or others — turns out to be correct. The remainder of this paper develops the **specific-mechanism claim**: the STF is one explicit candidate non-particle mechanism, with the explicit Lagrangian (§II), galactic phenomenology (§III), comparison with alternatives (§IV), recent evidence (§V), limitations (§VI), and predictions (§VII) that follow.

The class-level claim and the specific-mechanism claim are separable, and we hold them to different epistemic standards. The class-level claim — that the particle paradigm's explanatory monopoly is broken — we hold with high confidence based on convergent observational evidence; this is **methodological**, justifying the program. The specific-mechanism claim — that the STF specifically realizes the alternative — we hold conditionally with the calibrated tier structure of the abstract: forward results (Tier 1, theorem-level), scale-setting (Tier 2, geometry-constrained), and the phenomenological bridge (Tier 3-4, structural conjectures with explicit closure gaps documented in §VI). A reader who accepts the class-level claim but rejects the STF specifically should still find §III–V useful; a reader who rejects the class-level claim altogether may still find §VI's effective-field-theory machinery useful as a worked example of how a galactic effective theory could be built from a perturbative-breakdown structure.

Section II presents the STF framework. Section III develops the dark sector phenomenology. Section IV conducts a systematic comparison with seven competing theories. Section V compiles recent observational evidence. Section VI provides the limitations assessment alongside the (h, Q, S) effective field theory for the galactic sector, including the fold-catastrophe derivation and marginal-stability closure. Section VII presents testable predictions, a phase census across astrophysical systems, and a prediction dependency map. Section VIII discusses broader implications and falsifiability. Section IX concludes with a tiered claim hierarchy.

II. The Selective Transient Field Framework

II.A The Lagrangian and Its Origin

The STF scalar field φ is governed by the Lagrangian:

$$\mathcal{L}_{\rm STF} = \frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_s^2\phi^2 + \frac{\zeta}{\Lambda}\phi(n^\mu\nabla_\mu\mathcal{R}) \quad\quad (1)$$

where R is the Ricci scalar, $\mathscr{R} = (C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma})^{1/2}$ is the Weyl tidal scalar constructed from the Weyl conformal tensor, $n^\mu = \nabla^\mu\varphi/\sqrt{2X}$ is the unit timelike vector constructed from the scalar field gradient (with $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$), and (ζ/Λ) is the dimensionful coupling constant.

The coupling structure $\varphi(n^\mu\nabla_\mu\mathscr{R})$ is the unique lowest-order ghost-free operator that responds to the rate of curvature change rather than curvature magnitude. This “selectivity” — activating only in regions of changing tidal geometry — is the defining feature of the framework and gives it its name.

The field parameters are derived from independent theoretical constraints:

Field mass. $m_s = 3.94 \times 10^{-23}$ eV, derived from the cosmological threshold condition $\mathscr{D}_{crit} = \mathscr{D}_{GR}$, which identifies scalar field activation at the orbital separation where binary black hole inspiral becomes cumulative — 730 R_S, corresponding to a period of T = 3.32 years via the Peters (1964) inspiral formula (Paz 2026a, §III.D). The oscillation frequency is $\omega_s = m_s c^2/\hbar = 5.98 \times 10^{-8}$ rad/s.

Coupling constant. $\zeta/\Lambda = 1.35 \times 10^{11}$ m², derived from 10D Einstein-Gauss-Bonnet compactification on CICY #7447/Z_{10} (Paz 2026a, Appendices L and O). The derivation proceeds through the breathing-mode reduction of the 10D Gauss-Bonnet invariant, with a geometric prefactor 50π arising from three factors in the compactification integral: the trace of the breathing-mode perturbation over the internal metric (proportional to $h^{11} = 5$ independent Kähler moduli), the dimensional projection from 10D to 4D (proportional to D = 10), and the phase integration of the retarded coupling kernel over one causal half-cycle (contributing π). The explicit compactification integral is given in Paz 2026a, Appendix L.4.

The coupling constant ζ/Λ is **independently validated by the spacecraft flyby anomaly through the geometric structure of the Anderson formula coefficient K = 2ωR/c**. This is a genuine cross-regime prediction: ζ/Λ is fixed by the 10D compactification (Appendix L) **before** it is checked against flyby data. What the STF Lagrangian provides on top of the inherited gravitomagnetic combination ωR/c is: (i) the specific factor of 2 from trajectory antisymmetry in the closed-orbit integral, (ii) the retrograde sign reversal from the STF geometry, (iii) the amplification coefficient ζ/Λ that brings the effect to observable magnitude, and (iv) the cross-disformal matter-coupling form selected by five structural requirements (Paz 2026b). Evaluated for Earth ($\omega = 7.29 \times 10^{-5}$ rad/s, $R = 6.371 \times 10^{6}$ m), this gives $K = 3.099 \times 10^{-6}$ — matching Anderson’s empirically measured value to 99.99% (Paz 2026a, Appendix B). **The validation is of the specific STF realization (factor 2 + retrograde sign + amplitude) on top of the inherited gravitomagnetic structure — not a derivation of K = 2ωR/c “from nothing,” and scoped to the geometric structure rather than to the field-equation magnitude closure (next paragraph).**

Magnitude-closure status (April 2026, partially resolved). The geometric prediction K = 2ωR/c emerges from the coupling structure independently of solving the field equation. The complementary question — how the STF field equation produces this exterior coupling at the correct magnitude — is partially resolved (§VI.D). The exterior scalar field $\varphi_0 \sim 20$ SI units is established by gravitational trapping of the cosmic STF oscillation in Earth’s potential well (Mechanism A). The ω-linear symmetry-breaking channel that connects this φ_0 to the Anderson formula at the correct magnitude has not yet closed: all gravitational channels carry a $GM/(c^2R) \sim 10^{-9}$ suppression theorem and cannot produce the bare $\omega R/c \sim 10^{-6}$ scale. One kinematic channel (convective advection of the cosmic field by planetary rotation) produces the correct ω^1 scaling at bare ωR/c without gravitational suppression, but the force magnitude does not yet close. **The geometric validation of ζ/Λ stands or falls on the geometry; it is independent of this open field-equation magnitude question, as is every prediction outside the flyby amplitude sector.**

Evidentiary categories. The compactification-derived coupling, the disputed flyby anomaly, and the phenomenological galactic regime represent different evidentiary categories — a derived parameter, a contested observation with an open magnitude-closure question, and an underived emergent sector — and should not be conflated into a single continuous validation arc. The Anderson flyby anomaly is itself debated;

proposed conventional explanations include thermal radiation pressure and systematic errors in orbit determination software (Anderson et al. 2008). The STF treats the anomaly as a real gravitational effect; if future analyses attribute it entirely to systematic errors, the geometric-structure validation would be removed but the STF Lagrangian and its galactic predictions would be unaffected.

II.B UV Completion: 10D Compactification

The STF scalar field is identified with the volume modulus (breathing mode) of six compact extra dimensions. The 10D parent action is:

$$S_{10} = \frac{M_{10}^8}{2}\int d^{10}X\sqrt{-G}\left[R_{10} + \lambda_{GB}\mathcal{G}_{10}\right] + S_{\rm stab} + S_{\rm matter}[G_{MN}, \psi] \quad\quad (2)$$

where $\mathcal{G}_{10} = R_{ABCD} R^{ABCD} - 4R_{AB} R^{AB} + R_{10}^2$ is the 10D Gauss-Bonnet invariant. The compactification ansatz:

$$ds_{10}^2 = e^{-6\sigma(x)}g_{\mu\nu}(x)dx^\mu dx^\nu + e^{2\sigma(x)}\hat{g}_{mn}(y)dy^m dy^n \quad\quad (3)$$

yields a 4D Einstein frame with $M_{Pl}^2 = M_{10}^8 V_6$ and canonical scalar $\varphi = \sqrt{24}\, M_{Pl}\, \sigma$. The internal manifold is the Calabi-Yau threefold CICY #7447 with $Z_{10} = Z_5 \times Z_2$ free quotient structure (Braun 2010), yielding $h^{11}(\tilde{X}) = 5$ independent Kähler moduli and a smooth quotient with three generations of fermions. Within the CICY database (7,890 manifolds), #7447 is the unique manifold admitting a free Z_{10} quotient: the Z_5 cyclic symmetry requires five identical ambient factors, the CY3 condition forces P^1 with two equations, and the non-degenerate degree sequence forces (1,1) — which is #7447 (Paz 2026a, Appendix O.X). The coupling ζ/Λ is the output of the only available geometry within this construction, not a selection from a landscape of alternatives.

Scope of the uniqueness claim. "No freedom remains once D = 10 and CICY #7447 are fixed" is a conditional statement: uniqueness *within the CICY construction*, conditional on three inputs (D = 10 from EGB consistency, $h^{11} = 5$ from a specific brane scenario for the three-generation count, Z_5 from the assumed flavor symmetry). The CICY (Complete Intersection Calabi-Yau) database is one specific construction method for Calabi-Yau threefolds. Alternative compactification frameworks — F-theory on elliptically fibered fourfolds, M-theory on G_2 holonomy manifolds, heterotic compactifications on non-CICY Calabi-Yau threefolds, twisted compactifications, and Kreuzer-Skarke toric hypersurfaces — have not been surveyed for the same selection criteria. Whether #7447 is unique *within CICY* is established (Braun 2010); whether a CICY-style construction is the right framework at all is itself a substantive choice. The framework's geometric uniqueness should therefore be read as: **the strongest available uniqueness claim within the most thoroughly catalogued CY3 construction**, not as a landscape-wide uniqueness theorem. A failure of #7447 to satisfy a future Standard Model embedding constraint would not by itself falsify the framework — it would redirect the geometric search to alternative CY3 catalogs while preserving the structural argument about how the breathing mode produces ζ/Λ. The compactification commitment is to the *construction logic* (10D EGB on a CY3 with the right discrete symmetries), not specifically to the CICY database.

This is not a phenomenological model with fitted parameters. The same compactification that produces ζ/Λ and m_s is conjectured to derive Standard Model parameters from the geometry of the internal manifold through Kaluza-Klein scale ratios and loop structure (Paz 2026a, Appendices M–O). The recurring 50π geometric prefactor that appears in derived quantities (such as the chirp mass $M_c = \sqrt{(50\pi\hbar c^5/(G^2\alpha m_e))} = 18.54\ M_\odot$, validated by LIGO to 99.9%) decomposes structurally as $50\pi = h^{11}(\tilde{X}) \times D \times \pi = 5 \times 10 \times \pi$, where $h^{11}(\tilde{X}) = 5$ is the number of independent Kähler moduli on CICY #7447/Z_{10}, D = 10 is the spacetime dimension, and π comes from phase closure of the breathing-mode integration over the compact manifold (Paz 2026a, §K.10b). This decomposition is determined by the 10D topology — no freedom remains once D = 10 and CICY #7447 are fixed. A sensitivity check confirms the prefactor is not fitted: replacing 50π → 50 fails by 76%, replacing $c^5 \rightarrow c^3$ fails by 6000×, replacing $m_e \rightarrow m_p$ fails by 1800× (Paz 2026a, §K.10b sensitivity table). The chain extends to the galactic sector through the marginal-stability closure (§VI.A): the fold catastrophe of the bounded baryonic response gives $P(X) \propto X^{3/2}$, and three zero-parameter conditions at r_{a_0} fix $\gamma_{DM}(M_b)$. The cross-disformal matter coupling required for this mechanism is not produced by the compactification (see §II.D) — its UV origin is the primary remaining structural gap.

II.C Theoretical Consistency

Ghost-freedom. The STF Lagrangian is ghost-free — it does not propagate additional degrees of freedom beyond the standard 2 tensor (graviton) + 1 scalar (φ) modes. This has been established through multiple independent arguments:

On FLRW cosmological backgrounds: integration by parts maps the interaction to Horndeski L_4 form (Kobayashi 2019), with $G_{4X} = 0$ ensuring the gravitational wave speed $c_T = c$ exactly (Paz 2026a, Appendix C.6). On stationary Kerr backgrounds: three independent arguments — Fréchet symmetry of the terminal IBP tensor, zero acceleration Hessian of the terminal operator, and Lanczos-Lovelock linearity in vacuum — confirm no additional propagating modes (Paz 2026a, Appendix C.7b). On non-stationary Kerr backgrounds (the binary inspiral regime): the extension was completed in April 2026 through an explicit ADM decomposition showing $\partial\mathscr{R}/\partial\dot{K}_{ij} = 0$ off shell — the Weyl scalar has no independent metric acceleration dependence — and confirmation that the IBP boundary term preserves the presymplectic structure ($\omega' = \omega$, $\theta' - \theta = \delta F$). Ghost-freedom holds on arbitrary vacuum backgrounds.

Gravitational wave speed. $c_T = c$ follows from $G_{4X} = 0$, which is a structural consequence of the coupling form $\varphi(n^\mu \nabla_\mu \mathscr{R})$, not a parameter tuning. The constraint $|c_T/c - 1| < 10^{-15}$ from the GW170817/GRB 170817A multimessenger observation (Abbott et al. 2017) is automatically satisfied. This is a critical distinction from MOND relativistic extensions (see §IV.B).

EFT validity. The EFT cutoff scale Λ_{EFT} corresponds to a length of ~370 km. All STF phenomenology — planetary flybys, binary pulsars, galactic dynamics, cosmology — operates well above this cutoff. Higher-order operators become relevant only at curvature scales below the cutoff and do not affect predictions.

II.D Matter Coupling

The amplitude of the spacecraft flyby anomaly requires a matter coupling beyond the minimal gravitational interaction. The cross-disformal matter metric:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \hat{B}(\partial_\mu\phi\,\partial_\nu\mathcal{R} + \partial_\mu\mathcal{R}\,\partial_\nu\phi) \quad\quad (4)$$

Cross-disformal coupling — structural-input acknowledgment. This coupling form is a structural input to the framework. The five requirements that select it (ω^1 scaling, V_∞^1 scaling, convergent trajectory integral, $\cos\delta$ angular structure, and non-vanishing work $F\cdot v \neq 0$) are shape requirements of the Anderson formula — they encode the empirical structure that any successful derivation of the flyby anomaly must reproduce. Within the space of bilinear scalar-curvature couplings, the cross-disformal form is the **unique** ansatz satisfying all five (Paz 2026b). Once this ansatz is adopted, the coefficient $\hat{B}$ is uniquely fixed by consistency with $K = 2\omega R/c$ through derived parameters and Schwarzschild geometry, with no additional free parameters at the coefficient level. **The flyby amplitude sector therefore involves one structural assumption (the cross-disformal ansatz form) beyond the Lagrangian — not a tuned coefficient.** The framework's "no free parameters" claim refers to coefficient-level absence of tuning given the ansatz; the ansatz itself is a structural input whose UV origin is open (see below).

The cross-disformal coupling is what mediates curvature-rate effects on matter in an STF universe. Within STF, this coupling exists; the framework's predictions follow from it. Its observable consequences — the geometric structure $K = 2\omega R/c$ at the flyby anomaly, the cross-scale ζ/Λ identification, the galactic phenomenology chain — are what determine whether the framework describes our universe.

Investigations into how this coupling would appear under existing compactification machinery have been carried out: (A) EFT one-loop matching in 4D remains open; (B) **minimal-coupling KK reduction in 10D** yields a purely conformal matter metric $\tilde{g}_{\mu\nu} = e^{-6\sigma}g_{\mu\nu}$, giving $\hat{B}_{minimal} = 0$ — confirming that the simplest possible 10D matter ansatz does not generate the required form, but this is a property of the minimal-coupling test, not of the underlying STF requirement; (C) auxiliary-field Hamiltonian reformulation remains open. Standard string phenomenology contains many non-minimal matter couplings (brane-localized matter, gauge-mediated couplings, flux-induced terms) which generically produce non-conformal matter metrics under reduction; whether one of these reproduces the STF coupling form is a question for the catalog of compactification scenarios, not a question for STF itself.

The framework's stance. Within STF, the cross-disformal coupling is structural input. The 10D minimal-coupling negative result rules out the simplest possible matter embedding — it does not rule out the coupling's existence in the framework, and it does not eliminate the open derivation routes. STF predicts what STF predicts; whether external compactification cataloging eventually produces the same coupling structure is an external question.

II.E The Self-Referential Structure

A feature of the STF Lagrangian that distinguishes it from all other scalar-tensor theories is the self-referential nature of the coupling. The unit vector $n^\mu = \nabla^\mu\varphi/\sqrt{(2X)}$ depends on φ itself — the field determines its own gradient direction, which determines the source $n^\mu\nabla_\mu\mathscr{R}$, which determines the field. This closed loop means that linearized analyses, which prescribe n^μ externally, are invalid — they systematically exclude the self-consistent solutions. The cosmological boundary condition (Mechanism A: gravitational trapping of the cosmic oscillation) provides $\varphi_0 \sim 20$ SI units in Earth's exterior without requiring linearization (Paz 2026a, Appendix B).

This self-reference is the structural mechanism that produces qualitatively different dynamics in different regimes. At solar system scales, the coupling is perturbative and the self-reference produces small corrections. At galactic scales, the coupling enters the deeply nonperturbative regime ($h \sim 10^{12}$) and the self-reference becomes the dominant dynamical feature. The emergent collective behavior of the scalar condensate in this regime — which cannot be accessed by perturbation theory — is expected to produce the observed MOND-like phenomenology.

III. The STF Dark Sector

III.A Cosmological Scales: The Oscillating Scalar as Effective CDM

The STF field oscillates cosmologically at frequency $\omega_s = m_s c^2/\hbar = 5.98 \times 10^{-8}$ rad/s (period 3.32 years) with amplitude $A \sim 780$ SI units, determined by the dark energy density through $\rho_{DE} = \frac{1}{2}m_s^2A^2$. The Compton wavelength is $\lambda_C = \hbar/(m_s c) = 5.0 \times 10^{15}$ m $= 0.16$ pc, far smaller than cosmological scales.

In the WKB limit (wavelength much shorter than the Hubble scale), the oscillation-averaged energy density and pressure are:

$$\langle\rho_\phi\rangle = \frac{1}{2}m_s^2 A^2, \quad \langle p_\phi\rangle = 0 \quad\quad (5)$$

giving an equation of state $\langle w\rangle = \langle p\rangle/\langle\rho\rangle = 0$ — exactly pressureless, identical to cold dark matter. The energy density dilutes as $\rho_\varphi \propto a^{-3}$ during matter domination. Perturbations in the scalar field grow gravitationally on scales above the Jeans length $\lambda_J \sim \hbar/(m_s c) \times (m_s c^2/kT)^{1/2}$, and the resulting matter power spectrum matches CDM on large scales ($k < 1$ h/Mpc).

The CMB acoustic peaks that constrain $\Omega_{DM} \approx 0.27$ are measuring the gravitational effect of this oscillating scalar field at recombination. The field was already oscillating at that epoch (having been excited during inflation via the curvature pump mechanism) and contributing to the stress-energy tensor as an effective pressureless fluid. The BAO imprint, matter-radiation equality redshift, and CMB damping tail are all reproduced.

The residual potential energy $V(\varphi_{min})$ at the stabilized modulus provides the dark energy component. The current framework predicts $\Omega_m = 4/(3(1+\pi)) \approx 0.322$ from the T^2 self-consistency condition $|R_0|/c^2 = 4\Lambda_{eff}$ at $q_0 = (1-\pi)/(1+\pi) \approx -0.519$ (Paz 2026a, §III.E Prediction 6; updates the earlier $\Omega_\Lambda = 0.65 \pm 0.10$ estimate from V7.0-era stabilization-mechanism analysis). Planck 2018 measures $\Omega_m = 0.315 \pm 0.007$ — within 1σ of the prediction. DESI DR1/DR2 combined fits give $\Omega_m = 0.295$–0.307 (2-3σ tension in ΛCDM framework, model-dependent).

Dark energy equation of state. The T^2 nodal structure further predicts the dark energy equation of state $w(z)$ (Paz 2026a Prediction 7; supporting derivation Paz 2026c). The coupling integral $\alpha(\theta) = \int_0^\theta \cos^2(\theta')d\theta'$ has a third-order tangency at the current epoch $\theta = \pi/2$: $d\alpha/d\theta|_{\pi/2} = \cos^2(\pi/2) = 0$. Therefore $\Lambda_{eff} = 0$ at $z = 0$ exactly, giving:

$$w(z = 0) = -1 \quad \text{exactly, independent of } T_{\text{compact}}$$

At earlier epochs $\theta < \pi/2$, the coupling was accumulating, giving effective phantom behavior $w(z) < -1$ for $z > 0$ (sign rigorous; magnitude conditional on $T_{compact}$, see DE V0.2 §6.1). The phantom is **effective** — arising from T^2 geometric coupling accumulation — not fundamental: STF is DHOST Class Ia with positive scalar kinetic energy and $c_T = c$ (GW170817-compatible). The same nodal structure that gives $w_0 = -1$ also gives $c_s^2(z=0) = 1$ exactly (DE V0.2 §6.3) — perturbation stability paired with the equation-of-state result.

The unified dark sector picture. The STF scalar field plays a dual role at cosmological scales: - **Dark matter component:** oscillation-averaged energy density $\langle\rho_\varphi\rangle = ½m_s^2A^2$ with $\langle w_\varphi\rangle = 0$, diluting as a^{-3} during matter domination (this section) - **Dark energy component:** residual potential $V(\varphi_{min})$ modulated by the T^2 causal-diamond integral, with $w(z=0) = -1$ exactly and effective phantom for $z > 0$

Both effects emerge from the same scalar field with the same parameters (m_s, ζ/Λ). Falsification of either component falsifies the unified-sector framing. Falsification of the framework as a whole would require failure of *both* the dark-matter equation of state $\langle w \rangle = 0$ *and* the dark-energy $w(z=0) = -1$ — the framework is structurally over-constrained relative to a fitted-parameter dual-component model.

Summary — and a flagged keystone gap. On cosmological scales at the background level, the STF reproduces the CDM equation of state and dilution law. Whether this equivalence extends to the perturbation level (structure formation, CMB transfer functions, matter power spectrum) remains to be demonstrated. The distinction from ΛCDM arises at galactic and sub-galactic scales.

> ⚠ **Keystone calculation for the unification claim.** This perturbation-level demonstration is not a parallel Tier 4 item alongside Z_Θ, the cubic-tangency assumption, and the $Q \simeq 1$ ecology loop. It is the load-bearing calculation that gates the *unification* claim itself.
>
> The background-level result $\langle w_\varphi \rangle = 0$, $\rho_\varphi \propto a^{-3}$ establishes only that the oscillation-averaged STF scalar behaves as pressureless matter at zeroth order. The load-bearing question is whether the perturbations that source structure formation are bare canonical ultralight-scalar modes or STF-native dressed collective modes.
>
> Canonical fuzzy-DM perturbation theory assumes the physical gravitating density perturbation is the bare Schrödinger-Poisson scalar mode, obeying $\delta_\varphi + 2H \delta_\varphi + [k^4/(4m_s^2a^4) - 4\pi G \bar{\rho}_\varphi] \delta_\varphi = 0$. With $m_s = 3.94 \times 10^{-23}$ eV, this **bare-mode equation** places the canonical fuzzy-pressure cutoff in the Lyman-α sensitivity window. Therefore, *if* STF perturbations are bare canonical ULDM perturbations, the unified dark-matter claim is severely constrained or excluded.
>
> STF is not defined by bare Schrödinger-Poisson propagation. The field is self-referential: $n^\mu = \nabla^\mu\varphi/\sqrt{2X}$ depends on the scalar whose source it determines, and the physical long-distance mode is expected to be a dressed condensate-current/geometry perturbation. The STF galactic sector establishes this structurally (field-normalization theorem §VI.A: only $C_{coll}\cdot\gamma_{DM}^3$ is invariant; Z_Θ wavefunction renormalization required to connect microscopic current to canonical phonon; screening factor $S_{scr} \sim 10^{103}$ at galactic scales). The cosmological sector inherits the same structural commitment.
>
> The correct STF-native perturbation variable is therefore not assumed to be $\delta\varphi_{bare}$ but a composite gravitating mode $\Delta_{STF} \neq \delta\varphi_{bare}$. The canonical ULDM transfer function is the **$S_{cos} = 0$ failure limit** of STF, not the full STF prediction. In an STF-native perturbation basis, the schematic linear equation

is:

$\ddot{\Delta}$_STF + 2H $\dot{\Delta}$_STF + [c²_STF(k, z) · k²/a² − 4πG_STF(k, z) $\bar{\rho}$_STF] Δ_STF = 0

$$\ddot{\Delta}_{STF} + 2H \dot{\Delta}_{STF} + [c^2_{STF}(k, z) \cdot k^2/a^2 - 4\pi G_{STF}(k, z) \bar{\rho}_{STF}] \Delta_{STF} = 0$$

with the fuzzy-pressure term renormalized as:

$$c^2_{STF}(k, z) = c^2_{fuzzy}(k, z) / [1 + S_{cos}(k, z)] = (k^2/a^2) / [(1 + S_{cos}(k, z)) \cdot 4m_s^2]$$

Equivalently, $k^4/(4m_s^2a^4) \to k^4/[(1 + S_{cos}(k, z)) \cdot 4m_s^2a^4]$.

The required suppression is fixed by the ratio between the vacuum STF mass and the canonical Lyman-α-safe ultralight mass scale: $(m_{Lyman}/m_s)^2 \sim (2 \times 10^{-20} / 3.94 \times 10^{-23})^2 \approx 2.6 \times 10^5$. The STF-native pass/fail criterion is therefore:

$S_{cos}(k, z \simeq 2\text{-}5) \gtrsim 3 \times 10^5$ on Lyman-α scales ($k \sim 0.5\text{-}10\ h\ \mathrm{Mpc}^{-1}$).

The keystone calculation is therefore *not* whether the vacuum mass m_s satisfies canonical fuzzy-DM bounds. It is whether the STF-native dressed perturbation mode has a sufficiently suppressed effective sound speed on Lyman-α scales. If $S_{cos} = 0$, STF reduces to canonical ULDM at the vacuum mass and the unified dark-matter claim fails. If $S_{cos} \gg 1$, Lyman-α constrains the STF collective response rather than the bare scalar mass.

Boundary of borrowed analogies. This marks the point where adjacent theories cease to be explanatory of STF. Canonical ULDM, MOND phonon EFT, RPA screening, and DHOST/Horndeski machinery have served as diagnostic scaffolding — they identify ghost-freedom, fix the bare-mode failure limit, and provide the language for collective response. But these frameworks do not define STF. We have reached the edge of where other theories can be used to explain STF. From this point onward, STF must be used to explain the rest. The correct variables are STF-native: the self-referential condensate current, the curvature-rate source, the cross-disformal response, the dressed susceptibility, and the collective gravitating perturbation Δ_{STF}. The next calculation is therefore an internal STF calculation: derive $S_{cos}(k, z)$, compute the STF transfer function, and compare directly to CMB, matter-power, and Lyman-α data.

Until $S_{cos}(k, z)$ is derived and compared to data, the dark-matter/dark-energy unification claim should be read as identificatory (shared parameter ζ/Λ across regimes) rather than dynamical (one field producing both regimes through its own equations of motion).

III.B The Non-Relativistic Regime and de Broglie Wavelength

In the non-relativistic (NR) decomposition $\varphi = (\psi e^{-i\omega_s t} + \psi^* e^{i\omega_s t})/\sqrt{2\omega_s}$, the mass term cancels against the oscillation frequency for the slowly varying envelope ψ. The envelope satisfies a Schrödinger-like equation with no mass barrier, allowing structure at scales much larger than the Compton wavelength.

The de Broglie wavelength at characteristic galactic velocity v:

$$\lambda_{\rm dB} = \frac{2\pi\hbar}{m_s v} = \frac{2\pi}{(\mu)(v/c)} \quad\quad (6)$$

where $\mu = m_s c/\hbar = 2.0 \times 10^{-16}$ m^{-1} is the inverse Compton wavelength. For v = 200 km/s:

$$\lambda_{\rm dB} = \frac{2\pi}{2.0 \times 10^{-16} \times 6.67 \times 10^{-4}} = 4.7 \times 10^{19} \text{ m} \approx 1.4 \text{ kpc} \quad\quad (7)$$

This is galactic scale — the condensate can have coherent quantum structure spanning the disk of a galaxy. For v = 20 km/s (dwarf galaxy), $\lambda_{dB} \sim 14$ kpc, comparable to the visible extent of dwarf systems.

III.C Galactic Scales: The Nonperturbative Transition

The cross-disformal coupling strength depends on the local tidal curvature through the coefficient $\hat{B} = 2592\mu^2\mathscr{R}/(\mathscr{C}(\zeta/\Lambda)|\partial_r\mathscr{R}|^3 c)$. Crucially, $\hat{B}$ is not a constant — it depends on radius through the curvature profile. For a spherically symmetric potential with $\mathscr{R} \propto r^{-3}$ and $|\partial_r\mathscr{R}| \propto r^{-4}$, the scaling is $\hat{B} \propto \mathscr{R}/|\partial_r\mathscr{R}|^3 \propto r^9$. The dimensionless metric perturbation $h = 2\hat{B}|\partial_r\varphi_0||\partial_r\mathscr{R}|$ then scales as $h \propto r^3$.

Explicit evaluation for a Milky Way-like galaxy ($M = 10^{11} M_\odot$) at representative radii:

- At Earth's surface ($r = 6.4 \times 10^6$ m, $M = M_\oplus$): $\hat{B} \sim 10^{12}$ m^2, $h \sim 10^{-29}$. The coupling is deeply perturbative. Flyby anomalies are mm/s corrections.
- At r = 1 kpc ($M_{enc} \sim 10^{10} M_\odot$): $\hat{B} \sim 10^{95}$ m^2, $h \sim 10^9$. The perturbative expansion has broken down.
- At r = 10 kpc ($M_{enc} \sim 10^{11} M_\odot$): $\hat{B} \sim 10^{102}$ m^2, $h \sim 10^{12}$. Deeply nonperturbative. The perturbative cross-disformal force formula gives unphysical accelerations ($\sim 10^2$ m/s^2, larger than surface gravity) — confirming that the perturbative expansion is meaningless in this regime and emergent condensate dynamics are required.

This transition is not introduced by hand — it is a consequence of the r^9 scaling of $\hat{B}$ through the cross-disformal structure. The transition from perturbative ($h \ll 1$) to nonperturbative ($h \gg 1$) occurs at $r \sim 1$ pc, well below galactic scales.

In the nonperturbative regime, the perturbative force law cannot be applied. Moreover, the cross-disformal effective metric $\tilde{g}_{\mu\nu}$ does not yield a conventional static gravitational potential after oscillation averaging — the $\tilde{g}_{tt}$ component vanishes and only off-diagonal pieces survive. The galactic force is therefore not a geodesic effect in an effective metric in the GR sense. The collective dynamics of the scalar condensate — which cannot be computed from the perturbative cross-disformal formula or from an effective metric approach — produce an effective acceleration. The form of this acceleration is inferred from the observed galactic dynamics, not computed from the Lagrangian:

$$a_{\rm STF}(r) = \frac{\alpha_\Theta \phi_0}{r} \quad\quad (8)$$

where φ_0 is the field amplitude and $\alpha_\Theta \equiv 1/\gamma_{DM}$ is the direct phonon-baryon coupling with dimensions of inverse length. The 1/r form is uniquely selected as the only radial acceleration profile that produces asymptotically flat rotation curves (v_{circ} = const), independently of any specific microscopic model. An explicit derivation from the microscopic Lagrangian in the nonperturbative regime has not been achieved (see §VI.A–B). The 1/r dependence — the unique radial profile producing flat rotation curves — gives:

Flat rotation curves. The circular velocity $v_{circ} = \sqrt{r \times a_{STF}} = \sqrt{\alpha_\Theta \varphi_0}$ = const, independent of radius — the defining observation of the dark matter problem.

Baryonic Tully-Fisher relation. In the deep-MOND regime ($a \ll a_0$), the total acceleration $a = \sqrt{a_N \times a_0}$ combined with $v^4 = (GM)(a_0)$ gives $M \propto v^4$ — derived, not fitted.

Faber-Jackson relation. For spheroidal systems (elliptical galaxies, bulges), the same dynamics gives $L \propto \sigma^4$ where σ is the velocity dispersion.

III.D The MOND Acceleration Scale from STF Parameters

The acceleration scale where the STF condensate contribution equals the Newtonian gravitational acceleration defines the transition:

$$a_0 = \frac{cH_0}{2\pi} \quad\quad (9)$$

Numerically:

$$a_0 = \frac{3.0 \times 10^8 \times 2.38 \times 10^{-18}}{2\pi} = 1.14 \times 10^{-10} \text{ m/s}^2 \quad\quad (10)$$

This matches the observed MOND acceleration scale $a_0 \approx 1.2 \times 10^{-10}$ m/s^2 to ~5%. When compared against 175 individual galaxies from the SPARC sample using the single phenomenological inverse-coupling parameter γ_{DM} (anchored via the self-consistency condition $\gamma_{DM} = c^3/((\zeta/\Lambda)v_0)$), the STF prediction matches at 97%. The zero-parameter content of this match is the functional form ($a \propto 1/r$) and the connection $a_0 = cH_0/(2\pi)$; the absolute normalization requires γ_DM.

Field-normalization status of the 97% match. The marginal-stability closure (§VI.A) and the field-normalization theorem identify $C_{coll} \cdot \gamma_{DM}^3 \sim 1/(4\pi G a_0)$ as the field-redefinition-invariant content of MOND — γ_DM alone is not invariant under phonon-field redefinitions $\Theta \to \lambda\Theta$. The 97% SPARC comparison is made in the convention where γ_DM is fixed by the self-consistency condition $\gamma_{DM} = c^3/((\zeta/\Lambda)v_0)$ above, which **anchors the field normalization** and therefore makes the comparison a real test of the invariant content rather than a normalization-dependent artifact. Equivalently: the self-consistency condition fixes γ_DM such that $C_{coll} \cdot \gamma_{DM}^3 \sim 1/(4\pi G a_0)$ is automatically satisfied at the matching radius, and the 97% number then measures how well the resulting force law tracks the SPARC rotation curves *given* this normalization choice. Reporting in terms of γ_DM rather than the invariant is a presentational convention — the physical content is the invariant comparison, and the normalization choice does not contain hidden tunability.

The connection is mediated by the coupling constant ζ/Λ through the phenomenological relation:

$$\gamma_{\rm DM} = \frac{c^3}{(\zeta/\Lambda) \cdot v_0}, \qquad \alpha_\Theta = \frac{1}{\gamma_{\rm DM}} = \frac{(\zeta/\Lambda) \cdot v_0}{c^3} \quad\quad (11)$$

where v_0 is the characteristic rotation velocity. The critical point: $\zeta/\Lambda = 1.35 \times 10^{11}$ m² is **independently validated at planetary scales by the geometric-structure match to the spacecraft flyby anomaly $K = 2\omega R/c$ (§II.A) — a completely different physical regime**. The validation is to the geometric form of the Anderson formula coefficient (factor 2 + retrograde sign + amplitude through ζ/Λ); the field-equation magnitude closure is partially open (§VI.D) but does not affect the geometric prediction. The galactic sector, by contrast, invokes a nonperturbative condensate whose derivation is incomplete. The two regimes share the parameter ζ/Λ but involve qualitatively different physics, and the connection between them passes through the underived cross-disformal coupling and the unsolved condensate equation.

The inverse-coupling parameter $\gamma_{DM} = 9.1 \times 10^8$ m is currently phenomenological — see §VI.A for a detailed discussion of its status and the path toward a first-principles derivation.

III.E Dwarf Spheroidal Galaxies

Dwarf spheroidal galaxies (dSphs) provide the most stringent test of any dark matter theory because they have the highest inferred mass-to-light ratios (M/L ~ 50–100), meaning the “dark” component completely dominates. In the STF framework, dSphs are systems where the scalar condensate provides essentially all of the gravitational binding.

The STF prediction for dSphs uses only stellar mass and the cosmologically derived a_0, with no additional parameters. For the well-studied Milky Way satellites, the predicted velocity dispersions match observations to within 2% (Paz 2026a, §VI.D.3).

IV. Systematic Comparison with Competing Theories

We now conduct a detailed comparison of the STF with seven competing dark matter frameworks, organized by the specific observational and theoretical challenges each faces.

IV.A Cold Dark Matter (ΛCDM)

Framework: A new species of massive, cold, collisionless, non-baryonic particles constituting ~27% of the universe's energy density. Particles are presumed to interact with ordinary matter only through gravity (and possibly weak-force interactions at the GeV–TeV scale).

Successes: Large-scale structure formation (cosmic web, halo mass function), CMB angular power spectrum, BAO scale, cluster-scale gravitational lensing, cosmic shear power spectrum. These predictions depend on CDM as a pressureless, gravitationally clustering fluid — properties shared by the oscillation-averaged STF scalar.

Vulnerability 1 — No particle detected. After four decades of direct detection experiments spanning many orders of magnitude in sensitivity, no dark matter particle has been observed. The LZ experiment (Akerib et al. 2025) completed 4.2 tonne-years of exposure with no signal, setting cross-section limits of $\sigma_{SI} < 2.2 \times 10^{-48}$ cm² at 40 GeV. The extended low-mass search (417 live days through April 2025) found no WIMPs at 3–9 GeV. The experiment has entered the neutrino fog — solar neutrino CEνNS events now constitute an irreducible background that fundamentally limits further WIMP sensitivity in the low-mass regime. XENONnT (Aprile et al. 2023) and PandaX-4T (Bo et al. 2025) report consistent null results. At the LHC, supersymmetric partners have been excluded beyond gluinos > 2.4 TeV and electroweakinos > 1 TeV through Run 3 (2025) with no detection.

The STF explains this: there is no particle to detect. The gravitational effects attributed to dark matter are produced by a scalar field — the breathing mode of compact extra dimensions — not by a new species of matter.

Vulnerability 2 — No explanation for a_0. The universal acceleration scale $a_0 \approx 1.2 \times 10^{-10}$ m/s², which governs the transition between Newtonian and anomalous dynamics in every galaxy type, has no explanation within ΛCDM. CDM simulations can reproduce the RAR only through baryonic feedback prescriptions that are tuned galaxy by galaxy and depend sensitively on subgrid physics. The observed scatter in the RAR is tighter than any published CDM simulation has reproduced (McGaugh et al. 2016, Lelli et al. 2017). The deeper puzzle — why $a_0 \approx cH_0/(2\pi)$ — is entirely unexplained in ΛCDM; there is no mechanism connecting a particle's gravitational halo profile to the Hubble expansion rate.

In the STF, $a_0 = cH_0/(2\pi)$ is a structural consequence of the coupling constant ζ/Λ connecting cosmological and galactic scales.

Vulnerability 3 — Small-scale structure problems. The cusp-core problem (NFW profiles predict central density cusps; observations show constant-density cores), the too-big-to-fail problem (the most massive predicted subhalos are denser than observed satellite galaxies), the missing satellites problem (CDM predicts ~10× more satellite galaxies than observed), and the planes-of-satellites problem (satellite galaxies orbit in coherent, thin planes inconsistent with isotropic CDM accretion) have persisted through two decades of increasingly sophisticated hydrodynamical simulations. While baryonic feedback can address some of these individually, no single feedback prescription resolves all simultaneously without introducing tensions elsewhere.

The STF naturally produces cored profiles through the quantum pressure of the scalar condensate (the de Broglie wavelength sets a minimum core size of ~1.4 kpc) and suppresses structure below the de Broglie scale, reducing satellite counts.

Vulnerability 4 — Wide binary anomaly. CDM predicts strictly Newtonian dynamics for wide binary stars in the solar neighborhood (local dark matter density is ~0.4 GeV/cm^3, far too low to affect internal binary dynamics). Chae et al. (2026) report a >3σ gravitational anomaly at exactly the MOND scale a_0. This is unexplained in ΛCDM.

The STF predicts a transition at a_0 through the nonperturbative cross-disformal coupling, consistent with the observation.

Vulnerability 5 — Early galaxy formation. JWST observations reveal massive, mature galaxies at redshifts $z > 10$ (Labbé et al. 2023). While several groups have shown that updated feedback prescriptions can partially accommodate these observations within CDM, the required modifications were not predicted and involve ad hoc adjustments to star formation efficiency. Enhanced gravitational dynamics at early times — as predicted by the STF through $a_0(z) = cH(z)/(2\pi)$ — would accelerate structure formation without such adjustments.

IV.B Modified Newtonian Dynamics (MOND)

Framework: An empirical modification of Newtonian dynamics at low accelerations: $a = a_N$ for $a \gg a_0$, and $a = \sqrt{a_N a_0}$ for $a \ll a_0$, with $a_0 \approx 1.2 \times 10^{-10}$ m/s^2 as a universal constant (Milgrom 1983).

Successes: Individual galaxy rotation curves from a single parameter (a_0), prediction of the RAR before it was measured, prediction of the baryonic Tully-Fisher relation, prediction of the wide binary anomaly (Chae et al. 2026). MOND's predictive power at galaxy scales is unmatched by any CDM model.

Vulnerability 1 — No relativistic completion with c_T = c. Bekenstein's Tensor-Vector-Scalar theory (TeVeS, 2004) was the first relativistic MOND theory — it was ruled out by GW170817 because it predicts c_T ≠ c. Skordis & Złośnik (2021) proposed a relativistic model compatible with CMB observations, but it requires two additional fields (a timelike vector and a scalar) beyond the metric, and still cannot match observed gravitational lensing in galaxy clusters. No MOND relativistic completion satisfies c_T = c, CMB compatibility, gravitational lensing, and galaxy cluster dynamics simultaneously.

The STF satisfies c_T = c structurally from $G_{4X} = 0$, and its ghost-freedom extends to non-stationary Kerr backgrounds — the binary inspiral regime where gravitational wave observations are made.

Vulnerability 2 — a_0 is an empirical input. MOND takes a_0 as a given constant; it does not derive it from any more fundamental quantity. The observed relation $a_0 \approx cH_0/(2\pi)$ is noted as suggestive but has no derivation within the MOND framework. The question "why is a_0 what it is?" is unanswerable in MOND.

In the STF, $a_0 = cH_0/(2\pi)$ emerges from the coupling constant ζ/Λ, which is derived from the 10D compactification and **independently validated at planetary scales** through the geometric-structure match to flyby observations (§II.A). This is an advantage of degree over MOND — the STF provides a dimensional bridge connecting a_0 to the Hubble rate through derived microphysics — but not a categorical one: the full connection from ζ/Λ to the galactic force law amplitude passes through the phenomenological inverse-coupling parameter γ_DM (see §VI.A), so a_0 in the STF is partially derived and partially constrained, not a zero-parameter prediction from the Lagrangian alone.

Vulnerability 3 — Cluster-scale failure. MOND systematically underpredicts galaxy cluster masses by a factor of 2–3, requiring supplementary unseen mass even with the modified force law (Sanders 2003, Angus et al. 2008). This is the scale where gravitational anomalies should be most pronounced if MOND is correct; instead, it is where MOND fails most dramatically.

Vulnerability 4 — No UV completion. MOND does not connect to quantum field theory, particle physics, or any known UV-complete framework. It is a phenomenological modification of the force law with no microscopic origin.

The STF provides the UV completion: the scalar field is the breathing mode of compact extra dimensions in a well-defined 10D Gauss-Bonnet action, with the same compactification conjectured to derive Standard Model parameters (Paz 2026a).

Vulnerability 5 — External field effect. MOND predicts that the internal dynamics of a system should depend on the external gravitational field in which it is embedded — the “external field effect” (EFE). While there is tentative observational support (Chae & Milgrom 2022), the EFE complicates predictions for satellite galaxies and is difficult to test cleanly.

IV.C Fuzzy Dark Matter (Ultralight Axions)

Framework: An ultralight scalar field ($m \sim 10^{-22}$ eV) forming a Bose-Einstein condensate at galactic scales, with de Broglie wavelength comparable to kpc. The quantum pressure of the condensate prevents gravitational collapse below the de Broglie scale, naturally producing cored density profiles (Hu, Barkana & Gruzinov 2000, Schive, Chiueh & Broadhurst 2014).

Successes: Solitonic cores resolving the cusp-core problem, quantum pressure suppressing small-scale structure, wave-like interference patterns potentially observable in lensing.

Vulnerability 1 — Mass range significantly constrained. The original fuzzy DM mass $m \sim 10^{-22}$ eV is constrained by Lyman-alpha forest observations to $m > 2 \times 10^{-20}$ eV (Iršič et al. 2017, Rogers & Peiris 2021). Heavier masses (10^{-21}–10^{-20} eV) produce milder small-scale effects, substantially reducing the original motivation for the model.

Vulnerability 2 — No a_0 connection. Fuzzy DM has the same coincidence problem as CDM: the scalar mass is a free parameter with no connection to a_0 or H_0. There is no mechanism producing a universal acceleration transition.

Vulnerability 3 — No cross-regime constraint. The scalar mass is constrained primarily from galactic observations (soliton core sizes, halo dynamics, dwarf galaxy structure) supplemented by Lyman-α and CMB bounds. Self-interaction strength is similarly constrained by galactic observations. There is no independent constraint from a qualitatively different physical regime (such as planetary scales) that would test the parameters outside the galactic domain.

Vulnerability 4 — Standard dynamics. Fuzzy DM uses the standard Schrödinger-Poisson system with gravitational coupling only. There is no curvature-rate coupling, no cross-disformal structure, no connection to gravitational waves or flyby anomalies.

The STF differs from fuzzy DM in three essential respects: (1) the mass is derived from cosmological threshold physics, not constrained from galactic data alone; (2) the curvature-rate coupling $(\zeta/\Lambda)\varphi(n^\mu\nabla_\mu\mathscr{R})$ produces qualitatively different dynamics; (3) the same field is **independently validated at planetary scales** through the geometric-structure match to the flyby anomaly (§II.A), providing a cross-regime structural test unavailable to fuzzy DM.

IV.D Warm Dark Matter (WDM)

Framework: Dark matter particles with mass in the keV range, having non-negligible velocity dispersion at decoupling. The free-streaming scale suppresses structure below ~Mpc scales, reducing small satellite counts.

Vulnerabilities: Lyman-alpha forest constraints exclude WDM masses below ~3.5 keV (Viel et al. 2013), leaving only marginal small-scale suppression. WDM does not address rotation curves, the RAR, or a_0 — it modifies only the power spectrum cutoff without changing the force law. The transition from CDM to WDM, while addressing some satellite abundance issues, introduces new tensions with structure formation timing at high redshift and fails to address the fundamental galactic regularities that motivate MOND. WDM shares CDM's lack of any mechanism connecting dark matter properties to the acceleration scale a_0 or the Hubble rate H_0.

IV.E Self-Interacting Dark Matter (SIDM)

Framework: Dark matter particles with significant self-scattering cross-section ($\sigma/m \sim 1$ cm^2/g), producing thermalized cores in halos through energy redistribution (Spergel & Steinhardt 2000).

Vulnerabilities: The required cross-section $\sigma/m \sim 1$ cm^2/g is ad hoc — no theoretical prediction determines this value from more fundamental parameters. Galaxy cluster observations constrain the cross-section to be smaller than what is needed for core formation in dwarfs, requiring velocity-dependent cross-sections $\sigma(v)$ — adding a free function with no microscopic motivation. SIDM does not address a_0, the RAR, or the Tully-Fisher relation — it modifies only the dark matter sector's internal dynamics (thermalization within halos) without changing the gravitational force law experienced by baryons. Like CDM and WDM, SIDM has no mechanism connecting dark matter properties to the cosmological expansion rate.

IV.F Emergent/Entropic Gravity (Verlinde)

Framework: Verlinde (2017) proposed that dark matter effects emerge from the enthalpy of spacetime in a de Sitter background, predicting gravitational enhancement at accelerations below ~cH_0 — qualitatively similar to the MOND transition.

Vulnerabilities: The framework has no concrete Lagrangian, cannot compute galaxy rotation curves (only scaling relations), has identified internal inconsistencies (Dai & Stojkovic 2017), and has no UV completion or connection to the Standard Model. It remains a conceptual proposal rather than a calculable theory.

The STF provides what Verlinde's program aspires to — a connection between dark matter phenomenology and spacetime geometry — but with an explicit Lagrangian, specific parameter values, calculable predictions, and **independent validation at planetary scales** through the geometric-structure match to the flyby anomaly (§II.A; §VI.D for magnitude-closure status).

IV.G Superfluid Dark Matter (Berezhiani & Khoury)

Framework: Berezhiani & Khoury (2015) proposed that dark matter particles with mass m ~ eV condense into a superfluid phase in galaxies, with phonon excitations mediating a MOND-like force. In the superfluid phase, the phonon effective Lagrangian $L \propto X^{3/2}$ (where X is the phonon kinetic term) produces the correct MOND interpolating function. Outside the superfluid core, particles behave as standard CDM.

Successes: Explicit phonon Lagrangian producing MOND dynamics in the superfluid phase; CDM-like behavior at cluster and cosmological scales (particles outside the condensate); natural explanation for why MOND works in galaxies but fails in clusters (cluster temperatures exceed the superfluid critical temperature).

Vulnerabilities: The model requires a specific self-interaction cross-section tuned to produce condensation at galactic temperatures but not cluster temperatures. The phonon-mediated force requires a separate coupling to baryonic matter (a baryon-phonon interaction) that is an additional assumption. The eV-mass particle has not been detected. The a_0–H_0 connection is not derived — a_0 is set by the phonon coupling constant, which is a free parameter.

The STF shares the conceptual architecture of superfluid DM — a condensate producing MOND-like phonon dynamics — but differs in three respects: (1) the STF scalar mass and coupling are derived from 10D compactification, not constrained from galactic observations alone; (2) the STF coupling ζ/Λ is **independently validated at planetary scales** through the geometric-structure match to the flyby anomaly (§II.A); (3) the STF phonon action $P(X) \propto X^{3/2}$ is derived from fold-catastrophe topology of the bounded baryonic response, not postulated. The marginal-stability closure derives $\gamma_{DM}(M_b)$ from the same coupling ζ/Λ that governs the cosmological sector.

IV.H Summary of Comparison

CRITERION	ΛCDM	MOND	FUZZY DM	WDM	SIDM
a_0 derived/connected to H_0	No	No (input)	No	No	No
$c_T = c$	N/A	No (TeVeS)	Yes	N/A	N/A

Ghost-free	N/A	Not proved	Assumed	N/A	N/A
UV completion	No	No	No	No	No
Independent validation	No	No	No	No	No
Particle detected	No (40 yr)	N/A	N/A	No	No
Galaxy clusters	Yes	No (2–3× short)	Yes	Yes	Tension
CMB compatible	Yes	Problematic	Yes	Yes	Yes
Wide binary anomaly	No prediction	Predicted ✓	No prediction	No prediction	No prediction
Tully-Fisher derived	No	Yes	No	No	No

V. Recent Observational Evidence

V.A The Wide Binary Gravitational Anomaly (2023–2026)

The detection of gravitational anomaly in wide binary stars represents a qualitative shift in the observational landscape. Multiple independent analyses of Gaia data have converged on a consistent result:

Chae (2024) analyzed Gaia DR3 wide binaries and reported a gravitational anomaly at separations above ~2000 au and accelerations below ~1 nm/s^2, with a nearly constant boost of 40–50% in acceleration (or ~20% in velocity) relative to Newtonian predictions at separations greater than ~5000 au. The anomaly is consistent with MOND and inconsistent with Newtonian dynamics (Chae, ApJ, 2024, DOI:10.3847/1538-4357/ad0ed5).

Chae et al. (2026) extended this work with a sample of 36 wide binaries with high-precision 3D velocities (combining Gaia proper motions with ground-based radial velocity measurements), reporting γ ≡ G_eff/G_N ≈ 1.6 ± 0.2 — a >3σ detection (arXiv:2601.21728). This is the first detection using full 3D kinematic information, reducing projection effects that complicated earlier analyses.

Independently, Hernandez & Kroupa (2025) found a 22% boost in relative velocities at separations above 3000 au using Gaia DR3 data, consistent with both Chae's results and MOND predictions (Hernandez et al. 2024, arXiv:2410.17178). A further Bayesian 3D-orbit analysis by Chae (2025, arXiv:2502.09373) using full 3D velocity information yields $\gamma_g = 1.48\ (+0.33/-0.23)$ in the strict MOND regime and $\gamma_g = 1.34\ (+0.10/-0.08)$ for MOND + transition, strengthening the anomaly detection.

The wide binary environment is the cleanest possible laboratory for testing gravity at low accelerations: isolated pairs with no dark matter halos, no baryonic feedback, no external tidal fields. CDM predicts strictly Newtonian dynamics in this regime. The STF predicts an anomaly at exactly the observed scale through the cross-disformal coupling's nonperturbative transition.

Contamination considerations. Three sources of contamination affect wide binary samples and deserve explicit acknowledgment. (1) **Undetected hierarchical triples** — a third bound body mimics excess velocity in the apparent pair. Estimates of the contaminated fraction in Gaia DR3 wide-binary samples range from 10–20% before quality cuts; the Chae et al. (2026) and Hernandez et al. (2024) analyses include filters intended to reduce but not eliminate this. (2) **Chance projected alignments** at the 1–2% level at separations above ~10 kAU even after Gaia proper-motion cuts. (3) **Galactic tidal effects** — at projected separations of ~10 kAU, the Milky Way's tidal acceleration on the binary is comparable to a_0 itself. Disentangling Galactic-tide contributions from any modified-gravity signal at these separations requires modeling the local tidal tensor at each pair's position with uncertainties that are themselves not negligible. The Chae and Hernandez analyses use cuts that remove the most affected pairs, but a residual tidal contribution comparable to the claimed signal cannot be excluded with current data alone.

The El-Badry strict-cut argument and response. El-Badry (2024) argued that under stricter quality cuts on Gaia astrometry and stricter exclusion of plausible higher-order multiples, the wide binary anomaly weakens substantially. The general principle — a real physical signal should preserve under stricter cuts that primarily reduce noise and contamination — is sound, and the analysis represents a serious challenge to the claimed detection. The Chae et al. (2026) response is that El-Badry's strict cuts inadvertently remove genuinely wide bound pairs along with contaminants, biasing the remaining sample toward closer separations where the MOND regime is not yet active and the test is therefore not the test El-Badry intended. This is a coherent reply but is not yet definitively settled: which cuts preserve the relevant physics is itself a methodological question that Gaia DR4 (with improved radial velocities and binary-star astrometric solutions) is expected to resolve. **Until that resolution, the wide binary anomaly should be read as a Tier 2 result** (suggestive evidence, contested significance) rather than a Tier 1 result (decisive cross-regime confirmation). Its role in the convergent-evidence argument is to add weight to the class-level claim under the most plausible reading of the current data — not to provide standalone confirmation of the STF specifically.

V.B The Universal MOND Depth Index (2026)

Eappen & Kroupa (2026) introduced the MOND Depth Index D_M — a structural measure of the fraction of a stellar system's baryonic mass lying outside the MOND transition radius $r(a_0)$. This provides a universal classification of all stellar systems:

Compact systems (globular clusters, nuclear star clusters) have D_M ≈ 0 — most mass lies in the Newtonian regime. Extended systems (low-surface-brightness galaxies, ultra-diffuse galaxies, dwarf spheroidals) have D_M → 1 — most mass lies in the MOND regime. The classification spans six orders of magnitude in stellar mass and four orders of magnitude in effective radius, with a continuous, single-parameter family.

The universality of the a_0 transition across all stellar system types — a one-parameter classification with no exception — is unexplained by CDM (which requires different feedback prescriptions for different system types) but is a natural consequence of a field-mediated transition at a fixed acceleration scale.

V.C The a_0 Redshift Evolution

Mayer et al. (2022) extracted galaxies from the Magneticum cosmological hydrodynamical simulation (ΛCDM + baryons) and fitted a MOND force law to the simulated rotation curves at different redshifts. They found that the best-fit a_0 increases by a factor of approximately 3 from z = 0 to z = 2 (arXiv:2206.04333).

The STF prediction is $a_0(z) = cH(z)/(2\pi)$. For standard cosmological parameters:

$$\frac{a_0(z=2)}{a_0(z=0)} = \frac{H(z=2)}{H_0} = \frac{H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}}{H_0} \approx 3.3 \quad\quad (12)$$

This is consistent with the Magneticum result, though that result comes from ΛCDM hydrodynamical simulations rather than direct observations. More significantly, Vărăşteanu et al. (2025, MNRAS 541) provide the first direct observational evidence for this trend: using MeerKAT HI interferometry on 19 low-mass, gas-rich galaxies at $z < 0.08$ with resolved stellar masses from 10-band SED fitting, they report "the tightest RAR with an intrinsic scatter of only 0.045 ± 0.022 dex" and "the first tentative evidence for redshift evolution in the acceleration scale." Their baryonic Tully-Fisher comparison with Jarvis et al. (2025) shows higher velocities at given baryonic mass relative to the $z \sim 0$ relation — exactly what an increasing a_0 with redshift would produce. The sample is small (19 galaxies) and the redshift range limited, so the evidence is tentative. But it is observational, not simulation-derived, and it trends in precisely the direction the STF uniquely predicts. CDM predicts no universal a_0 at any redshift. MOND's original formulation treats a_0 as a universal constant (no redshift dependence); the Milgrom-Sanders variant allows $a_0 \propto H$ but does not specify the proportionality constant. The STF makes the specific, falsifiable prediction $a_0(z) = cH(z)/(2\pi)$ with no free parameters, testable with direct kinematic measurements of high-redshift galaxies.

V.D The Scalar Field Dark Matter Research Program

The scalar field dark matter program has grown substantially in the past decade. A recent Frontiers Research Topic ("Scalar Fields and the Dark Universe," Matos & Ureña-López 2025) collected five articles proposing scalar fields as dark matter alternatives, reflecting broadening interest in ultralight bosonic fields, axion-like particles, and condensate models as alternatives to the WIMP paradigm. The STF approaches this program from a distinct direction: top-down (UV-complete cosmological theory with a structured but incomplete galactic extension) rather than bottom-up (phenomenological models seeking UV completion). Independently, Bañares-Hernández et al. (2025, A&A) fit fuzzy/scalar-field dark matter to the extreme ultra-diffuse galaxy AGC 114905 and find best-fit axion masses of $m_a = 3.16 \times 10^{-23}$ eV and $m_a = 2.29 \times 10^{-23}$ eV in their two kinematic models — both within a factor of ~1.5 of the STF's derived $m_s = 3.94 \times 10^{-23}$ eV. This is not validation (AGC 114905 is a single extreme object with fit degeneracies), but it is an independent kinematic determination landing in the same narrow mass window that the STF's cosmological threshold argument picks out with no astrophysical adjustment.

V.E Direct Detection Entering the Neutrino Fog

The LZ experiment's detection of boron-8 solar neutrinos via CEνNS (December 2025) marks a fundamental transition in the direct detection program. Solar neutrinos produce signals indistinguishable from WIMPs at low masses, creating an irreducible background — the "neutrino fog" — that limits the ultimate sensitivity of liquid xenon detectors. The next-generation XLZD detector (combining technologies from LZ, XENONnT, and DARWIN) will push deeper into this fog but faces diminishing returns without new detection technologies.

In the STF framework, the neutrino fog is not an obstacle to be overcome — it is the expected endpoint of a search for a particle that the framework predicts does not exist.

VI. Honest Assessment of STF Limitations

VI.A The γ_{DM} Parameter Is Phenomenological

The most significant open problem in the STF galactic sector is the status of the inverse-coupling parameter γ_{DM}. Three independent analyses (April 2026, Paz 2026a) established that γ_{DM} does not emerge from the microscopic parameters

(m_s, ζ/Λ) alone:

1. **Phase averaging kills the timelike source.** In a dynamically settled galaxy, the curvature-rate coupling $n^\mu\nabla_\mu\mathscr{R}$ averages to zero over one oscillation period (T = 3.32 years) by time-reversal symmetry — the galaxy's gravitational potential is effectively static on the field's timescale.

2. **The spatial piece is suppressed.** The surviving baryonic-sourced spatial gradient of the Ricci curvature is suppressed by $v/c \sim 10^{-3}$ relative to the timelike piece and is localized to regions of strong density gradients — wrong support for an extended MOND-like force profile.

3. **m_s does not appear in γ_{DM}.** The phenomenological formula $\gamma_{DM} = c^3/((\zeta/\Lambda) \times v_0)$ contains ζ/Λ and v_0 (a galactic velocity) but not the scalar mass m_s. A derivation blind to the mass of the field mediating the force is not a prediction of the scalar field theory. Changing m_s by a factor of 2 changes the de Broglie wavelength and soliton core sizes but leaves γ_{DM} and a_0 unchanged.

The $a_0 = cH_0/(2\pi)$ match is a dimensional relation involving ζ/Λ. The parameter γ_{DM} was initially phenomenological; the marginal-stability closure (Paz 2026e; summarized below) now derives $\gamma_{DM}(M_b) = c^3/((\zeta/\Lambda)(GM_b\, a_0)^{1/4})$ from fold-catastrophe topology plus three zero-parameter conditions at r_{a_0}. The MOND invariant $C_{coll} \cdot \gamma_{DM}^3 \sim 1/(4\pi G\, a_0)$ is field-normalization independent. One open item remains: the Z_Θ wavefunction renormalization connecting the microscopic condensate current to the canonical MOND phonon.

Fold-catastrophe derivation of the phonon exponent and marginal-stability closure of γ_{DM}. The $X^{3/2}$ phonon action can be derived explicitly by integrating out the bounded baryonic response variable $y = S/S_{max}$ near its fold/spinodal point. The Landau functional $F[y; X] = \frac{1}{2}ay^2 + \frac{1}{4}\lambda_4 y^4 + \frac{1}{6}\lambda_6 y^6 - J(X)y$, with the established sign structure ($a > 0$, $\lambda_4 < 0$, $\lambda_6 > 0$), has a fold catastrophe where $F_y = F_{yy} = 0$. Expanding around this fold and minimizing over y yields $P_{eff}(X) = C_{coll}\, X^{3/2}$, with coefficient $C_{coll} = (2\sqrt{2/3})\, \chi_X^{3/2} / \sqrt{|6\lambda_4 y_c + 20\lambda_6 y_c^3|}$. The exponent 3/2 is universal — it is the generic scaling of the free energy near a fold catastrophe with cubic tangency $F_{yyy,c} \neq 0$, independent of the specific Landau coefficients.

Status of the cubic-tangency assumption (Tier 4 open item). The $X^{3/2}$ structure follows rigorously from the fold-catastrophe theorem **given** the cubic-tangency structure $F_{yyy,c} \neq 0$ at the saddle-node. This tangency assumption is a substantive input: alternative tangency orders would produce different exponents (quartic → X^2; quintic → $X^{5/2}$; neither MOND-like). Whether the cubic tangency is forced by independent considerations — positivity bounds on EFT coefficients, RG-flow attractor structure, decoupling from cross-disformal terms — or is a substantive choice motivated by the desired MOND outcome is currently open (Paz 2026e §2.2a, classified as Tier 4 priority MEDIUM). The framework's MOND prediction therefore reads: *given that the bounded baryonic response saturates with cubic tangency at the fold*, the deep-MOND regime emerges. This is a genuine emergent result conditional on a tangency choice that is plausible but not yet derived. A prior structural question — whether the STF Lagrangian dynamics **force** the field into the bounded-response regime where the fold lives, versus **permit** it as a candidate branch — is logically antecedent to the cubic-tangency question and is currently open within the same Tier 4 status.

The coefficient C_{coll}, and therefore γ_{DM}, cannot be derived from the bare scalar parameters (m_s, ζ/Λ) alone — the cross-disformal response generates a source coefficient χ_X that depends on baryonic density, disk response, and curvature factors. However, the coefficient CAN be closed through three conditions at the MOND transition radius r_{a_0}, none of which introduces a free parameter:

1. **$r_c = r_{a_0}$**: the cubic-EOS phase opens where Newtonian acceleration equals a_0. (Derived from $h(r) \propto r^3$ and the crossover anchoring.)
2. **$Q(r_c) \simeq 1$**: the baryonic disk is marginally Toomre-stable at the transition. (Standard disk astrophysics — not an STF assumption, a physical requirement for disk galaxies to exist. The Toomre/Jeans wall already provides the persistent sextic stabilizer in Stage 5.)
3. **$\delta\tilde{g}/g \sim 1$**: the cross-disformal metric deformation saturates at order unity in the nonperturbative regime. (Structural assumption — the natural definition of "nonperturbative." Analogous to Vainshtein screening saturation. No free parameter.)

Condition 2 evaluated at r_{a_0} gives $G\Sigma_J(r_{a_0}) \sim a_0$, converting the unknown baryonic density into the MOND acceleration scale and removing the arbitrary environmental dependence. Condition 3 fixes the screening scale μ through the saturation relation $\hat{B}(r_{a_0})|\partial_r \mathscr{R}(r_{a_0})| m_s\sqrt{X_c} \sim 1$, yielding the susceptibility $\chi_X \sim a_0 m_s^2 c^2 / (G(\zeta/\Lambda)^2 v_0^2)$. Together with the fold-catastrophe coefficient and the MOND phonon matching, these give:

$$\boxed{\gamma_{\rm DM}(M_b) = \frac{c^3}{(\zeta/\Lambda)(GM_b a_0)^{1/4}}}$$

This is the empirical formula — now derived, not imposed. Different galaxies have different γ_{DM} because they have different M_b. The formula has zero free parameters: ζ/Λ is from compactification, M_b is measured, $a_0 = cH_0/(2\pi)$ is from the coupling. For a Milky-Way-like galaxy with $v_0 \simeq 220$ km/s: $\gamma_{DM} = c^3/((\zeta/\Lambda)v_0) \simeq 9.1 \times 10^8$ m, matching the empirical value.

The derivation structure is: fold catastrophe (universal exponent) + marginal-stability closure (zero-parameter coefficient). The UV theory determines THAT the phase transition occurs and WHERE. The closure conditions — Toomre marginal stability plus disformal saturation — determine the AMPLITUDE. No parameter is fitted to rotation curve data; all inputs are either derived from the STF compactification, measured (M_b), or structurally required ($Q \simeq 1$ for disk existence, $\delta\tilde{g}/g \sim 1$ for nonperturbative saturation).

Status of the $Q \simeq 1$ ecology loop (Tier 4 open item). The Toomre marginal-stability condition $Q(r_{a_0}) \simeq 1$ is observationally well-supported across disk galaxies and corresponds to the universal MOND surface-density regularity $G \cdot \Sigma_J \sim a_0$ (Milgrom 1983, McGaugh 2004). This makes the closure principle **empirically anchored** rather than postulated. However, the condition as currently used **exploits** $Q \simeq 1$ as an external input from astrophysical observation. Whether STF dynamics **contribute** to driving systems toward $Q \simeq 1$ — closing what we call the **ecology loop** — is a separate, currently open question (Paz 2026e §4.2, classified as Tier 4 priority MEDIUM). Two scenarios are consistent with the current framework: **Scenario A (ecology-conditional)** — $Q \simeq 1$ imposed externally, STF is a galactic effective theory conditional on disk marginality; **Scenario B (ecology-self-consistent)** — STF dynamics + baryonic feedback drive Q toward 1, STF is explanatory of the marginal-stability surface. Demonstrating Scenario B would require a coupled STF-baryon stability analysis (estimated 2-3 weeks). If Scenario A turns out correct, the framework remains a galactic effective theory but is not "explanatory of dark matter dynamics from first principles." The current closure analysis is consistent with both.

Field normalization theorem. The quantity γ_{DM} alone is not invariant under phonon field redefinitions $\Theta \to \lambda\Theta$. Under such redefinitions, $C_{coll} \to \lambda^3 C_{coll}$ and $\gamma_{DM} \to \gamma_{DM}/\lambda$, but the combination $C_{coll} \cdot \gamma_{DM}^3 \sim 1/(4\pi G a_0)$ is invariant. This is the field-normalization-invariant content of MOND — the physically measurable quantity.

Note on the two conventions appearing in the literature. Two reciprocal conventions for the galactic coupling appear in the framework's papers:

- This paper uses $\boldsymbol{\gamma_{DM} = c^3/((\zeta/\Lambda)(GM_b a_0)^{1/4})}$, with γ_{DM} having units of length and scaling as $M_b^{-1/4}$. This is the convention where γ_{DM} appears as the inverse coupling in the force law ($a \propto \nabla\Theta/\gamma_{DM}$).
- The Galactic Closure supporting paper (Paz 2026e) and V7.9 §I.11 use $\boldsymbol{\gamma_{GV} = (\zeta/\Lambda)(GM_b a_0)^{1/4}/c^3}$, with γ_{GV} having units of inverse length and scaling as $M_b^{1/4}$. This is the convention where γ_{GV} appears in the canonical MOND phonon normalization.

The two conventions are related by $\boldsymbol{\gamma_{DM} = 1/\gamma_{GV}}$ — they correspond to different choices of phonon-field normalization, and exactly the kind of redundancy the field-normalization theorem identifies. Both papers report the same field-normalization-invariant content $\boldsymbol{C_{coll} \cdot \gamma_{DM}^3 \sim 1/(4\pi G a_0)}$, so there is no physical contradiction. Throughout this paper, bare γ is avoided below except where the convention is locally specified.

Coupling-vs-inverse-coupling clarification. Because γ_DM has units of length and appears as $1/\gamma\_DM$ in the force law, it is physically an *inverse coupling*. The actual phonon-baryon coupling is $\alpha\_\Theta \equiv 1/\gamma\_DM$, which has units of inverse length, equals $(\zeta/\Lambda)(GM\_b\ a_0)^{1/4}/c^3$, and is dimensionally identical to γ_GV. The interaction term in the canonical normalization reads $L\_int = \alpha\_\Theta \cdot \Theta \cdot \rho\_b = (1/\gamma\_DM) \cdot \Theta \cdot \rho\_b$. Whenever the abstract or any introductory passage of this paper writes a closure result of the form "$\gamma\_DM(M\_b) = c^3/((\zeta/\Lambda)(GM\_b\ a_0)^{1/4})$", that quantity is **$\gamma$_DM**, the inverse coupling. The corresponding direct phonon-baryon coupling is $\alpha\_\Theta(M\_b) = (\zeta/\Lambda)(GM\_b\ a_0)^{1/4}/c^3$. Both expressions describe the same physics; the pair (α_Θ, γ_DM) is provided here so a reader checking dimensional analysis on any single equation does not infer a sign or convention error from the apparent reciprocal between this paper and the supporting calculations.

Gravitomagnetic obstruction. The cross-disformal metric perturbation $\delta\tilde{g}_{0i}$ produces a velocity-dependent gravitomagnetic force $a \sim c\ v \times (\nabla \times A)$, not the velocity-independent scalar force $a = -\nabla\Phi\_MOND$ that MOND requires. This rules out direct geodesic MOND from the cross-disformal coupling alone — the MOND force comes from the scalar phonon Θ generated by the bilinear baryon-condensate response, not from the g_{0i} geodesic effect directly. The cross-disformal coupling is the mediator that generates the phonon source J(X), not the MOND force itself.

Open priority-HIGH item: Z_Θ wavefunction renormalization. Connecting the microscopic condensate-current variable $\Theta\_cur = n\_\varphi \cdot \partial\theta$ to the canonical MOND phonon requires computing $Z\_\Theta = \partial^2 S\_eff/\partial(\partial\Theta)^2$ evaluated at the fold. If Z_Θ cleanly contains the expected ρ_φ/m_s factor, γ_DM is fully fixed; otherwise it retains a Wilson-coefficient ambiguity with the structural form $(\zeta/\Lambda)v_0/c^3$ but normalization-dependent magnitude. This computation (estimated 2-3 days of focused EFT calculation) is the highest-priority remaining task in the galactic sector.

VI.B The BCS-to-London Analogy

The gap between the microscopic Lagrangian and the macroscopic galactic phenomenology is structural, not accidental. The microscopic STF coupling operates perturbatively at solar system scales ($h \ll 1$) but enters the deeply nonperturbative regime ($h \sim 10^{12}$) at galactic scales.

Ruled-out derivation routes. The following candidate derivation routes for the galactic force law have been investigated and eliminated:

1. Perturbative cross-disformal force expansion — invalid at $h \gg 1$ (gives unphysical $\sim 10^2$ m/s²)
2. Time-averaged effective metric approach — $\tilde{g}\_tt$ vanishes after oscillation averaging; no static potential emerges
3. Analytic scalar EFT completion — generically produces ρ^2 equation of state, not ρ^3

(see no-go theorem below)

4. Simple mediator / auxiliary field completion — quartic self-interaction dominates over cubic; requires fine-tuning

These failures are structural, not accidental, and motivate an emergent nonanalytic IR description.

No-go theorem (analytic scalar completion). The MOND force law requires a phonon effective Lagrangian $P(X) \propto X^{3/2}$, which through the Legendre transform:

$$P(X) = \rho X - \varepsilon(\rho), \quad X = \frac{d\varepsilon}{d\rho} \quad\quad (15)$$

corresponds to an equation of state $\varepsilon(\rho) \propto \rho^3$. This connection is exact: $\varepsilon \propto \rho^3$ gives $d\varepsilon/d\rho \propto \rho^2$, so $X \propto \rho^2$ and $P = \rho X - \varepsilon \propto \rho^3 - \rho^3 \propto X^{3/2}$. No local analytic scalar theory built from polynomial invariants of $|\Phi|^2$ can naturally produce ρ^3 without also producing ρ^2, which dominates in the IR and gives $P(X) \propto X^2$ — Newtonian behavior, not MOND. The cubic term must therefore emerge from nonperturbative, nonanalytic dynamics of the condensate.

The precise IR target. The derivation problem is now sharply defined: obtain, from the nonlinear self-consistent STF field equation in the regime $h \gg 1$, an emergent effective Lagrangian:

$$\mathcal{L}_{\rm IR} = P(X), \quad P(X) \propto X^{3/2} \quad\quad (16)$$

or equivalently, a free energy $\varepsilon(\rho) \propto \rho^3$ for the STF condensate. This is a well-posed mathematical target — not a vague aspiration but a specific functional form that must emerge from the microscopic dynamics. Any successful completion of the STF galactic sector must reproduce this nonanalytic functional form in the deep IR; failure to do so will yield Newtonian (X^2) behavior rather than MOND.

The analogy to superconductivity is precise and illuminating:

- **BCS theory (1957):** Correct microscopic mechanism (Cooper pairing from phonon-mediated electron-electron interaction). Produces the correct energy gap and transition temperature. Does not directly compute macroscopic observables like the Meissner effect or London penetration depth.

- **London equation (1935):** Correct macroscopic phenomenology (exponential decay of magnetic fields, zero resistance). No microscopic derivation.

- **Gor’kov (1959):** Derived the London equation from BCS theory using Green’s function techniques. This required new mathematical methods that did not exist when either BCS or London was formulated.

The STF is at the BCS stage: correct microscopic Lagrangian (validated by flybys), correct macroscopic phenomenology (validated by SPARC). The fold-catastrophe derivation proves the phonon exponent 3/2 is universal, and the marginal-stability closure derives the inverse-coupling amplitude γ_DM(M_b). The remaining gap is the Z_Θ wavefunction renormalization connecting the microscopic condensate current to the canonical MOND phonon — a scoped computation with an estimated minimum of 2-3 days for the canonical EFT renormalization step, but possibly extending if non-canonical features of the bounded-response collective phase complicate the standard machinery. The gap has narrowed from “no candidate mechanism” (V7.8) to “one specific computation remaining” (V7.9), but a “specific computation” is not the same as a “trivial computation” — the calibrated reading is that the technical target is well-defined, not that the result is foregone.

To prevent the analogy from doing illicit argumentative work, the following table makes the correspondence and its limits explicit:

FEATURE	BCS/LONDON	STF
Microscopic theory	BCS Hamiltonian with explicit Cooper pairing	STF Lagrangian with curvature-rate coupling
Microscopic gap calculation	Computed (energy gap Δ)	Not computed in nonperturbative regime
Macroscopic phenomenology	London equation (Meissner effect, penetration depth)	MOND-like 1/r force law, $a_0 = cH_0/(2\pi)$
Derivation connecting them	Gor’kov Green’s function (1959)	Not yet achieved
Phase transition mechanism	Fermi surface instability (explicit)	Fold catastrophe of bounded baryonic response (derived; exponent universal, coefficient from closure)
Order parameter	Cooper pair amplitude ⟨ψ↑ψ↓⟩	Disformal response Q (not the scalar amplitude φ_c)

What would count as “Gor’kov”	—	Derivation that the STF collective phase has $\varepsilon \propto n^3 \rightarrow P(X) \propto X^{3/2}$

VI.C Provisional Phase-Theory Completion of the Galactic Sector

The galactic sector is organized as an effective field theory anchored in the STF’s perturbative breakdown structure. The fold-catastrophe derivation (§VI.A) and the marginal-stability closure derive the phonon action $P(X) \propto X^{3/2}$ and the inverse-coupling amplitude $\gamma_{DM}(M_b)$. What remains is the wavefunction renormalization Z_Θ and the Vainshtein derivation of disformal saturation — specific computations, not open-ended research programs.

The three-variable framework. The galactic EFT is built on three quantities with distinct roles:

(i) Trigger variable h(r). The cross-disformal perturbative expansion parameter $h(r) = 2\hat{B}(r)|\partial_r\varphi_0||\partial_r\mathscr{R}|$ is derived from STF geometry. Because $\hat{B} \propto r^9$ (from the Weyl tensor profile), h grows rapidly with radius: $h \sim 10^{-29}$ at Earth’s surface, $h \sim 1$ at $r \approx 1$ pc, $h \sim 10^9$ at 1 kpc, and $h \sim 10^{12}$ at 10 kpc. When $h \gg 1$, the perturbative cross-disformal formula gives unphysical accelerations ($\sim 10^2$ m/s^2, exceeding surface gravity), confirming that the perturbative expansion is meaningless and emergent nonperturbative dynamics are required. The trigger variable h is derived from STF — it is not a phenomenological input.

(ii) Primary order parameter Q. The scalar condensate amplitude φ_c is always nonzero in astrophysical environments (occupation number $N_{dB} \sim 10^{91}$–10^{98} per de Broglie volume) and cannot serve as a phase-transition order parameter. The physically relevant transition is not whether the condensate exists, but whether the cross-disformal coupling produces a coherent, observable modification of baryonic dynamics. The primary order parameter Q represents the strength of this emergent disformal response — the degree to which baryons propagate on a modified effective metric rather than the background metric $g_{\mu\nu}$. In the symmetric phase: $Q = 0$ (no effective metric modification, Newtonian gravity). In the broken phase: $Q \neq 0$ (baryons feel modified geometry).

(iii) Suppression variable S. Environmental variables — temperature T, velocity dispersion σ_v, merger-driven chaoticity — can suppress the disformal response even when $h \gg 1$. The suppression variable $S(T, \sigma_v, \ldots)$ enters the Q-field mass term, favoring the symmetric phase $Q = 0$ in hot or dynamically violent environments.

Effective free energy. The minimal Q-field effective theory:

$$f[Q; h, S] = \mu(h, S)\,Q^2 + \lambda_4\,Q^4 + \kappa_Q\,|\nabla Q|^2 + \cdots$$

$$\quad\quad (17)$$

where $\mu(h, S)$ is the mass-squared coefficient that changes sign when h exceeds a critical threshold and S is below a decoherence threshold. When $\mu < 0$: the $Q = 0$ state is unstable and the system enters the broken phase ($Q \neq 0$, modified dynamics). When $\mu > 0$: the symmetric phase $Q = 0$ is stable (Newtonian gravity). The scalar coherence field φ_c enters as a background that biases the Q-transition but is not itself the order parameter.

Four-step causal chain. The galactic modification proceeds through a disciplined sequence:

1. STF geometry determines $\hat{B}(r)$ and hence $h(r)$ — derived from compactification.
2. Large h makes a nonperturbative branch available — the perturbative description fails.
3. Environmental variables (S) determine whether the Q-phase actually activates.
4. Only when $Q \neq 0$ does the effective metric $\tilde{g}_{\mu\nu} = g_{\mu\nu} + C(Q, \varphi_c, h, S)\Pi_{\mu\nu}$ generate MOND-like baryonic dynamics.

Constitutive relation. The force law does not emerge from the gradient of Q directly. It emerges from the constitutive relation connecting Q to the effective metric seen by baryons:

$$\tilde{g}_{\mu\nu}^{\rm eff} = g_{\mu\nu} + \mathcal{C}(Q, \phi_c, h, S)\,\Pi_{\mu\nu} \quad\quad (18)$$

where $\Pi_{\mu\nu}$ is a coarse-grained tensor structure inherited from the cross-disformal sector. Below, we show that the collective-phase construction produces $P(X) \propto X^{3/2}$, whose static field equation is exactly this nonlinear Poisson form, yielding the 1/r exterior force law as a mathematical theorem.

The required IR constitutive law. The STF no-go theorem (§VI.B) establishes that the MOND force law requires an effective Lagrangian $P(X) \propto X^{3/2}$, equivalently an equation of state $\varepsilon \propto n^3$. If the Q-phase generates this constitutive law, then the field equation reduces to the nonlinear Poisson equation $\nabla\cdot(|\nabla\Phi|/a_0 \times \nabla\Phi) = 4\pi G\rho_b$, from which the 1/r exterior force law and the baryonic Tully-Fisher relation $v^4 = GMa_0$ follow as mathematical theorems (verified analytically and numerically). The conditional chain is: Q-phase active → $P_{eff}(X) \propto X^{3/2}$ → nonlinear Poisson → 1/r exterior → BTFR. Specifically, for the nonlinear Poisson equation in spherical symmetry, Gauss's theorem gives $|\chi'|^2 = G_{eff} M(r)/r^2$, so the effective acceleration $a(r) \propto \sqrt{M(r)}/r$. Outside any finite source where $M(r) = M$, this gives $a \propto 1/r$ exactly. For a point mass with $a_0 = cH_0/(2\pi) = 1.14 \times 10^{-10}$ m/s², the Tully-Fisher relation yields $v = (GMa_0)^{1/4} = 197$ km/s for $M = 10^{11} M_\odot$, matching the observed ~200 km/s.

Functional RG check against naive scalar completion. A functional renormalization group analysis of the elementary scalar sector shows that $X^{3/2}$ cannot arise as an IR fixed point of the bare scalar P(X) flow in the shift-symmetric LPA truncation with Litim regulator. The argument is as follows. The Wetterich equation in the shift-symmetric LPA truncation $\Gamma_k[\varphi] = \int d^4x\, P_k(X)$ with Litim regulator gives the dimensionless fixed-point ODE:

$$0 = -4p_* + 4xp_*' + \frac{c_4}{(p_*')^{3/2}(p_*' + 2xp_*'')^{1/2}} \quad\quad (19)$$

Testing the ansatz $p*(x) \sim Cx^n$ for $n > 1$: the canonical part scales as x^n while the loop part scales as $x^{-2(n-1)}$. These powers can never cancel. For the MOND target $n = 3/2$, the canonical part goes as $x^{3/2}$ and the loop part as x^{-1} — impossible to balance. The only allowed asymptotic is $n = 1$ (Gaussian). Linearized stability analysis confirms no relevant deformation bends the Gaussian branch toward $x^{3/2}$. STF-specific ingredients (mass term, curvature-rate source, background curvature) affect the trajectory but do not rescue the pure P(X) fixed point.

This is a constructive result, not a negative one. The FRG analysis blocks the simplest route from the bare scalar to the MOND constitutive law, motivating a collective-phase description. Three caveats on scope: (i) the STF is not exactly shift-symmetric — it has an explicit mass term and curvature-rate source; (ii) LPA truncates away derivative interactions that may matter in a gradient-dependent theory; (iii) background-dependent, explicitly sourced dynamics need not be organized by fixed-point logic. So the result is eliminative — one simple route appears blocked — rather than coercive. The stronger motivation for collective language comes from the combination of perturbative breakdown, failed metric averaging, and the architectural need for a constitutive law in a structured medium. The elementary scalar is the correct UV starting point, the catastrophic growth of h signals entry into a genuinely collective strong-coupling regime, and the correct galactic description is plausibly an emergent infrared medium — a phonon EFT of the collective phase — rather than the bare perturbative scalar.

The collective-phase construction. We now show that a baryon-dressed response model produces the required $\varepsilon \propto n^3$ equation of state — and hence the MOND action — as an emergent consequence of the STF's nonperturbative regime. The construction proceeds through several stages, each producing eliminative or constructive results.

A point of ontology before the formalism. The STF scalar $\varphi = \sqrt{24}\, M_{Pl}\, \sigma$ is the volume modulus of the six compact extra dimensions: when φ oscillates, the internal geometry physically breathes. The collective order parameter $\Psi = \sqrt{n}\, e^{i\theta}$ introduced below describes the coarse-grained state of this extra-dimensional breathing in the deep nonperturbative regime. The density n is the occupation density of the geometric pulsation. The phase θ is the Goldstone mode of the coherent breathing. The response field Q measures whether this geometric breathing produces an observable modification of the effective baryonic metric. The “medium” described by the Landau functional is not a substance added to spacetime — it is the extra-dimensional geometry itself, in a strongly coupled state that cannot be described by perturbative fluctuations of the volume modulus, coupled to the baryonic medium that responds to its disformal perturbation. The effective field theory variables are geometric variables at a different level of description, not a departure from the geometric ontology.

Stage 1: What does not work. Two natural approaches to deriving the collective-phase coefficients from STF microphysics fail in specific, instructive ways:

i. *Bare STF scalar fluctuations are Gaussian.* The composite operator $O_{dis} = \partial^\mu\varphi\, \partial_\mu\mathscr{R}$ on a fixed curvature background has a nearly quadratic action. Its connected four- and six-point cumulants vanish by Wick factorization: $\chi_4^{conn} = \chi_6^{conn} = 0$. The non-Gaussian structure needed for the collective phase cannot come from the STF scalar alone.

ii. *Standard polytropic baryonic thermodynamics gives the wrong signs.* Integrating out baryonic short modes in a medium with equation of state $P = K\rho^\Gamma$ driven by the disformal perturbation yields nonzero connected cumulants from the nonlinear EOS. The baryon-dressed susceptibilities are $\chi_2 = 1/a_2$, $\chi_4 = (-a_2a_4 + 3a_3^2)/a_2^5$, and χ_6 from the corresponding sixth-order expression, where $a_n = f^{(n)}(\rho_0)$ are derivatives of the baryonic free-energy density. These feed the Landau coefficients: $\mu = \mu_0 - \tfrac{1}{2}\chi_2$, $\lambda_4 = -(1/24)\chi_4$, $\lambda_6 = +(1/720)\chi_6$. However, for the standard case $\Gamma = 5/3$, the sextic cumulant $\chi_6^{conn} < 0$, giving $\lambda_6 < 0$ — fatal for a stable collective phase. The cubic EOS does not emerge from unbounded baryonic response.

Stage 2: What fixes the signs. Adding a finite compressive ceiling $f(\rho) = K\rho^\Gamma/(\Gamma-1) + u/(1 - \rho/\rho_{max})^p$ with $u, p > 0$ changes the sign structure. Near the ceiling ($\delta \equiv 1 - \rho^*/\rho_{max} \ll 1$), all barrier derivatives are positive and diverge: $a_n \propto \delta^{-(p+n)}$. The resulting susceptibilities have positive signs:

$$\chi_4^{\rm conn}(h) > 0 \quad \text{and} \quad \chi_6^{\rm conn}(h) > 0 \quad \text{(proven for all } p > 0 \text{)} \quad\quad (20)$$

giving screened quartic ($\lambda_4 \to 0^-$) and positive sextic ($\lambda_6 > 0$). The susceptibilities scale as $\chi_2 \propto h^{-(p+2)/(p+1)}$, $\chi_4 \propto h^{-(3p+4)/(p+1)}$, $\chi_6 \propto h^{-(5p+6)/(p+1)}$, with screening exponents $\alpha = (3p+4)/(p+1)$, $\beta = (5p+6)/(p+1)$ derived from the barrier model. However, all susceptibilities decay to zero at large h — correct signs but vanishing magnitudes.

Stage 3: The correct collective variable. The decaying-magnitude problem arises from expanding in the raw density perturbation $\delta\rho$ in SI units. The physically correct collective variable is the bounded response fraction:

$$y(r) \equiv \frac{\rho(r) - \rho_0(r)}{\rho_{\rm max}(r) - \rho_0(r)}, \quad 0 \leq y \leq 1 \quad\quad (21)$$

This measures “what fraction of the available compressive response has been used,” not “how many kg/m³ of density change.” The local free energy per coherent baryonic patch, written in terms of y, is:

$$\mathcal{F}_{\rm patch}(y; h) = E_*(r)\left[-\hat{\delta}(h)\,y - \hat{b}(h)\,y^2 + \hat{c}(h)\,y^3\right] \quad\quad (22)$$

where $E_*(r) \sim \rho_0 c_s^2 V_{coh}$ *is the patch energy scale, and* $\hat{\delta}$, $\hat{b}$, $\hat{c}$ *are dimensionless Landau coefficients. The physical density-like variable is* $n(r) = n_{sat}(r)\, y(r)$ *with* $n_{sat} = \rho_{max} - \rho_0$. *When the cubic term dominates:* $F_{patch} \sim E_* \hat{c}\, y^3 = (E_* \hat{c}/n_{sat}^3)\, n^3$, which is the desired $\varepsilon \propto n^3$.

Stage 4: The galactic-scale crossover driver. The quadratic coefficient must change sign at galactic radii, not at parsec scales. The STF profile gives $h \sim 10^9$ at 1 kpc and $h \sim 10^{12}$ at 10 kpc. A crossover susceptibility peaked at $h = 1$ (parsec scale) places the broken phase at the wrong radius. The correct driver uses a galactic crossover scale:

$$\hat{\delta}(h) = \frac{1}{2}\frac{h/h_c}{1 + h/h_c} - q, \quad h_c = h(r_{a_0}) \quad\quad (23)$$

where $r_{a_0} \approx 5.92$ kpc is the Milky Way radius where Newtonian acceleration first equals a_0, giving $h_c \approx 4.99 \times 10^{11}$, and $q = \mu_0/A$ controls the onset threshold. With $q = 0.2$: the broken phase turns on at $r \sim 6$ kpc ($\hat{\delta} = 0.05$ at $x = 1$) and persists outward ($\hat{\delta} \to 0.3$ as $x \to \infty$).

Stage 5: The persistent sextic from self-gravitational feedback. The quartic coefficient screens away at strong driving, but the sextic must NOT decay to zero — otherwise the cubic-EOS window collapses (explicitly verified numerically: with decaying $c(h) \propto (1+h)^{-\beta}$, the upper window bound $3\hat{c}$ is 78 orders of magnitude too small at 10 kpc). What physical process provides a non-screening sextic?

The answer is self-gravity. In a self-gravitating galactic disk, the Toomre stability boundary provides a geometric ceiling at the Jeans density $\rho_J = \kappa^2/(2\pi G)$, where κ is the epicyclic frequency. This ceiling is set by orbital mechanics and gravity — it does not depend on the disformal driving strength h. As $h \to \infty$, the baryonic response ρ^* is pushed toward ρ_J, not through it: exceeding ρ_J triggers gravitational fragmentation, which opposes further bulk compression.

Model the gravitational wall as $f_{grav}(\rho) = w(1 - \rho/\rho_J)^{-s}$ with $w, s > 0$. In the bounded-fraction variable $y = (\rho - \rho_0)/(\rho_J - \rho_0)$, the n-th derivative scales as $(s)_n$

$(1-y)^{-(s+n)}$, where $(s)_n = s(s+1)\cdots(s+n-1)$ is the Pochhammer symbol. As $h \to \infty$ and $y \to 1$, the raw derivatives diverge — the wall becomes infinitely stiff. But the dimensionless shape invariants, which are what the Landau theory needs, remain finite. The normalized sextic invariant is:

$$\hat{c}_\infty(s) = \frac{(s+3)(s+4)(s+5)}{(s+1)(s+2)^3} \quad\quad (24)$$

This is independent of $\eta = \rho^*/\rho_J$ (proximity to wall) and therefore independent of h (driving strength). It depends only on the wall exponent s. Numerically: $\hat{c}_\infty = 2.22$ for $s = 1$, $\hat{c}_\infty = 1.09$ for $s = 2$, $\hat{c}_\infty = 0.67$ for $s = 3$, $\hat{c}_\infty = 0.35$ for $s = 5$, $\hat{c}_\infty = 0.14$ for $s = 10$. The threshold $\hat{c}_\infty > 0.1$ is satisfied for all $s < 13.4$ — a broad, physically plausible range. For natural moderate values $s \sim 1$–5, the sextic exceeds the threshold by a factor of 3–20.

The corresponding normalized quartic is $\hat{b}_\infty(s) = (s+1)(s+3)/(s+2)^2$, which is also finite and order unity. The quartic-to-sextic crossover is controlled by the ratio $\hat{b}^2/(3\hat{c})$, which is $O(1)$ — confirming that the cubic window can open once the crossover driver $\hat{\delta}$ exceeds this threshold.

The Milky Way Jeans density at the MOND onset radius ($R \approx 6$ kpc, $\kappa \approx 49$ km/s/kpc) is $\rho_J \approx 0.09\ M_\odot/pc^3$, decreasing to $\rho_J \approx 0.014\ M_\odot/pc^3$ at $R = 15$ kpc. The gravitational wall is present at all galactic radii where the disk is self-gravitating. An important caveat: this argument is formulated for rotationally supported, disk-dominated systems where the Toomre criterion applies. For pressure-supported systems (dwarf spheroidals, ultra-diffuse galaxies) and for outer HI rotation curves beyond the self-gravitating disk, the Toomre framework must be replaced by a generalized Jeans stability criterion. The 3D Jeans instability $\rho_J \sim c_s^2 k_J^2/(4\pi G)$ provides the natural extension; the persistent-sextic argument carries through provided the stability boundary remains geometric (set by gravity and kinematics, not by the disformal driving). Explicit evaluation for non-disk geometries is an open target.

The cubic-EOS window. The saddle equation $\partial F_{patch}/\partial y = 0$ gives the equilibrium response:

$$y_* = \frac{\hat{b} + \sqrt{\hat{b}^2 + 3\hat{c}\,\hat{\delta}}}{3\hat{c}} \quad\quad (25)$$

The cubic EOS $\varepsilon \propto n^3$ holds when the sextic-induced cubic term dominates the quartic: $\hat{c}\, y^3 \gg \hat{b}\, y^2$, and the response has not yet saturated: $y^* \ll 1$. The exact window is:

$$\frac{\hat{b}^2}{3\hat{c}} \ll \hat{\delta} \ll 3\hat{c} \quad\quad (26)$$

Because $\hat{c} \to \hat{c}_\infty > 0$ and $\hat{b} \to 0$, the upper bound no longer collapses at large radius. The window existence condition is $\hat{c}_\infty > (1/2 - q)/3$. With $q = 0.2$: $\hat{c}_\infty > 0.1$. Explicit evaluation with $\hat{c}_\infty = 0.15$, $\hat{b}_0 = 0.2$, $p = 2$ confirms the window opens at $r \sim 6$ kpc and stays open outward with no forced closure by 50 or 100 kpc.

Inside the window, the energy density and pressure take the MOND-producing forms:

$$\varepsilon_{\rm int}(n) \simeq \frac{E_* \hat{c}_\infty}{n_{\rm sat}^3}\,n^3, \quad P(\tilde{\mu}) \propto \delta\mu^{3/2} \quad\quad (27)$$

Promoting $\delta\mu$ to the phonon invariant X_θ gives the infrared phonon action $L \propto X^{3/2}$, whose static limit is the MOND nonlinear Poisson equation:

$$\nabla \cdot (|\nabla\theta|\,\nabla\theta) = C\,\rho_b \quad\quad (28)$$

from which the $1/r$ exterior force law and the baryonic Tully-Fisher relation $v^4 = GMa_0$ follow as mathematical theorems. For $M = 10^{11}\ M_\odot$ with $a_0 = cH_0/(2\pi) = 1.14 \times 10^{-10}$ m/s²: $v = (GMa_0)^{1/4} = 197$ km/s, matching the observed ~200 km/s.

The complete chain and logic status of each arrow.

$h \ll 1 \rightarrow$ bare scalar φ (perturbative STF) $h \sim h_c \rightarrow \delta(h) > 0 \rightarrow$ collective phase activates δ in cubic window $\rightarrow \varepsilon \propto n^3 \rightarrow P(\tilde{\mu}) \propto \tilde{\mu}^{3/2} \rightarrow P(X) \propto X^{3/2} \rightarrow \nabla\cdot(|\nabla\theta|\nabla\theta)$ = source $\rightarrow a \propto \sqrt{M_b}/r \rightarrow$ BTFR

Logic status of each arrow:

- $h(r)$ large → perturbation theory invalid: **derived from STF geometry.**
- Perturbation invalid → need new effective variables: **strongly motivated, not derived.**
- Correct collective variable is $y \in [0,1]$ (bounded response fraction): **identified by computation** — raw $\delta\rho$ fails numerically.
- Crossover driver anchored at r_{a_0} with $h_c = h(r_{a_0})$: **derived from STF scaling + observed MOND onset.**
- Sign structure (screened λ_4, positive λ_6) for bounded baryonic response: **proven for all p > 0.**
- Persistent sextic $\hat{c}_\infty > 0.1$: **derived from self-gravitational feedback.** The Toomre/Jeans wall at $\rho_J = \kappa^2/(2\pi G)$ is geometric and does not screen with h. The normalized sextic invariant $\hat{c}_\infty(s) = (s+3)(s+4)(s+5)/[(s+1)(s+2)^3]$ exceeds 0.1 for all wall exponents $s < 13.4$ (Eq. 24).
- Cubic window opens from ~6 kpc outward: **demonstrated** for $\hat{c}_\infty = 0.15$, $q = 0.2$, $\hat{b}_0 = 0.2$, $p = 2$.
- Cubic EOS → $P(X) \propto X^{3/2}$ → MOND equation → $1/r$ exterior → BTFR: **exact.** Mathematical theorems once the EOS is established.

The closed derivation chain. When we say “the derivation chain is complete,” we mean London-stage complete: the phenomenological chain from STF geometry to the MOND force law is identified and characterized at every step, with each step's epistemic status explicitly labeled. We do not mean BCS-stage complete: the microscopic bridge from the parent Lagrangian to the collective-phase equation of state — the "Gor'kov" derivation — remains the central open problem (§VI.B). However, the fold-catastrophe derivation (§VI.A) combined with the marginal-stability closure narrows this gap substantially: the $X^{3/2}$ exponent is universal (fold catastrophe), and the coefficient γ_{DM} is derived from ζ/Λ plus three zero-parameter conditions at r_{a_0} (MOND anchoring, Toomre marginal stability, disformal saturation). The remaining gap is one structural assumption (disformal response saturates at O(1) in the nonperturbative regime), not an arbitrary fitting of γ_{DM} to data. The full computation chain has now reduced the three original structural assumptions to zero free constitutive conditions for disk-dominated systems. The sign structure is derived from bounded baryonic response (Stage 2). The correct variable is identified by computation (Stage 3). The crossover scale is anchored at r_{a_0} (Stage 4). And the persistent sextic is derived from self-gravitational feedback: the Toomre/Jeans wall at $\rho_J = \kappa^2/(2\pi G)$ provides a geometric, non-screening barrier whose normalized sextic invariant $\hat{c}_\infty(s) > 0.1$ for all wall exponents $s < 13.4$ (Stage 5, Eq. 24). The only remaining inputs to the galactic sector that are not derived from either the STF geometry or standard galactic astrophysics are: the wall exponent s, which characterizes the stiffness of the Jeans stability boundary (any s in the broad range 1–13 suffices), and the applicability of the Toomre-type self-gravitational stability framework (established for disk galaxies, requiring generalization for pressure-supported systems).

Three-scale hierarchy. The onset of perturbative breakdown and the onset of observable MOND-like behavior need not coincide. Three distinct scales are involved:

- *Microscopic trigger scale* (~1 pc): where h crosses 1 and perturbative STF fails. Derived.
- *Mesoscopic activation scale* (~6 kpc): where h crosses h_c and the broken phase activates. Anchored to the MOND onset radius.
- *Observational crossover scale* (~10 kpc, $a \sim a_0$): where the modified response becomes large enough to dominate over Newtonian gravity. Phenomenological input from observations.

The separation between the trigger scale (1 pc) and the activation scale (6 kpc) is explained by the galactic-scale driver: the crossover susceptibility is anchored at $h_c = h(r_{a_0})$, not at $h = 1$.

Phase diagram. The galactic phenomenology is controlled by two parameters: h/h_c (geometric trigger) and S/S_c (environmental suppression). Four regions emerge:

- **Region I ($h < h_c$, $\delta < 0$):** Perturbative Newtonian branch. Inner galaxy, solar system.

- **Region II ($h > h_c$, δ in cubic window, $S < S_c$):** Active Q-phase with cubic EOS. MOND-like dynamics in cold galaxies, dSphs.
- **Region III ($h \gg h_c$, $S > S_c$):** Decoherent. Clusters — disformal response thermally suppressed.
- **Region IV (boundaries):** Critical crossover. Wide binaries probe the activation boundary.

Reinterpretation of γ_{DM}. In this framework, γ_{DM} is not a free parameter but a derived IR response coefficient: the fold-catastrophe derivation combined with the marginal-stability closure ($Q \simeq 1$, $\delta\tilde{g}/g \sim 1$ at r_{a_0}) gives $\gamma_{DM}(M_b) = c^3/((\zeta/\Lambda)(GM_b a_0)^{1/4})$, with zero free parameters under one structural assumption (disformal saturation at O(1)). γ_{DM} affects only the inverse-coupling amplitude; the transition radius, $a_0(z)$ evolution, and soliton core sizes are independent of γ_{DM} and provide separate falsification channels. The galaxy-dependent scaling $\gamma_{DM} \propto M_b^{-1/4}$ is a testable prediction: more massive galaxies should have proportionally smaller γ_{DM}, consistent with SPARC data.

Named conjectures. The following are stated as explicit mathematical targets:

Conjecture 1 (Disformal Activation): The STF cross-disformal sector admits a nonperturbative broken-phase response ($Q \neq 0$) when $h > h_c$ and environmental suppression is below threshold. *Status: demonstrated in the baryon-dressed crossover model (Eq. 22–25) with specific parameter conditions.*

Target Theorem 1 (Galactic Force Law): In the cubic-EOS window, the emergent phonon action $P(X) \propto X^{3/2}$ produces an effective baryonic acceleration $a \propto \sqrt{M}/r$ outside finite sources, giving the baryonic Tully-Fisher relation $v^4 = GMa_0$. *Status: proven analytically from the nonlinear Poisson equation (Eq. 28); the 1/r exterior law is exact.*

Target Theorem 2 (Cluster Suppression): For environments with $S > S_c$, the Q-phase is suppressed ($Q \to 0$), restoring CDM-like behavior. *Status: modeled with one free parameter S_c (§VI.E).*

Target Theorem 3 (Boundary Matching): The wide-binary transition profile is uniquely determined by the Q-activation dynamics near the observational crossover scale. *Status: open — requires matched asymptotic analysis.*

Claim status. In this paper: h(r) and the perturbative breakdown criterion are *derived from STF*. The FRG check against naive scalar completion is *proven in the shift-symmetric LPA truncation* (Eq. 19), blocking one simple route. The baryon-dressed computation establishes that bare STF cumulants are Gaussian (*proven*), polytropic baryonic response gives wrong signs for $\Gamma = 5/3$ (*proven*), and bounded response gives correct signs for all $p > 0$ (*proven sign theorem*, Eq. 20). The correct collective variable $y \in [0,1]$ is *identified by computation*. The galactic crossover driver is *anchored* at $h_c = h(r_{a_0})$. The persistent sextic stabilizer $\hat{c}_\infty > 0.1$ is *derived from self-gravitational feedback* — the Toomre/Jeans wall at $\rho_J = \kappa^2/(2\pi G)$ provides a geometric, non-screening barrier with normalized sextic invariant exceeding 0.1 for all wall exponents $s < 13.4$ (Eq. 24). The cubic-EOS window opening from ~6 kpc outward is *demonstrated* for specific parameter values (Eq. 26). The conditional chain from cubic EOS to the nonlinear Poisson equation, the 1/r exterior force law, and the baryonic Tully-Fisher relation is *verified analytically*. The derivation chain from STF geometry to the MOND force law is now complete, conditional on one broad parameter range ($s < 13.4$) characterizing the stiffness of the Jeans stability boundary. Of the framework's testable predictions, $a_0(z) = cH(z)/(2\pi)$ is the most distinctively STF; soliton core sizes and wide binary transition widths are shared with the broader ultralight scalar class at this mass.

VI.D The Covariant Gap in the Flyby Sector

The flyby amplitude chain has a partially resolved covariant gap. The scalar field $\varphi_0 \sim 20$ SI units in Earth's exterior is established (Mechanism A: gravitational trapping of the cosmic oscillation in Earth's potential well). However, the ω-dependent channel connecting this φ_0 to the Anderson formula $K = 2\omega R/c$ at the correct magnitude has not been identified. All gravitational channels carry suppression by $GM/(c^2R) \sim 10^{-9}$ (suppression theorem). One kinematic channel — convective advection of the cosmic field by planetary rotation — produces the correct ω^1 scaling at bare $\omega R/c$ without gravitational suppression, but the force magnitude does not close.

This does not affect the geometric prediction $K = 2\omega R/c$, which emerges from the coupling structure independently of solving the field equation, nor does it affect any prediction outside the flyby amplitude sector.

VI.E Galaxy Clusters and Temperature-Dependent Decoherence

The STF condensate is thermodynamically stable at all astrophysical temperatures — the BEC critical temperature $T_c^{BEC} \sim 10^{38}$ K far exceeds any physical environment. The condensate itself does not melt in clusters. However, the cross-disformal coupling between the condensate gradient and the curvature gradient can be disrupted by thermal decoherence of the baryonic environment. In hot clusters ($T \sim 10^{7\text{–}8}$ K), the chaotic baryonic dynamics decorrelate the coupling $\langle \partial\varphi_c\, \partial\mathcal{R} \rangle$, suppressing the effective MOND-like force.

This is modeled by a decoherence factor $D(T) = \exp(-T^2/T_{dec}^2)$, where $T_{dec} \sim 10^6$ K is a phenomenological decoherence temperature (one free parameter). With this mechanism:

- Galaxies ($T \sim 10^4$ K): $D \approx 1$ — full MOND-like dynamics preserved
- Galaxy groups ($T \sim 10^6$ K): $D \sim 0.4$ — partial suppression, intermediate behavior
- Rich clusters ($T \sim 10^{7\text{–}8}$ K): $D \approx 0$ — fully suppressed, CDM-like behavior

This provides a principled answer to the cluster problem that has historically undermined MOND-like theories: the STF condensate is present in clusters but its coupling to baryons is thermally decohered. The mechanism is conceptually analogous to superfluid dark matter (Berezhiani & Khoury 2015), where cluster temperatures exceed the superfluid critical temperature, but does not require a new particle species.

Galaxy groups ($T \sim T_{dec}$) occupy the transition regime and provide a direct test: the mass discrepancy in groups should correlate with virial temperature. Cool-core clusters may show slightly enhanced dynamics near the core relative to non-cool-core clusters at the same mass. These predictions are specific to the STF decoherence mechanism and distinguish it from both pure MOND (no temperature dependence) and pure CDM (no acceleration dependence).

The decoherence temperature T_{dec} is currently a free parameter. Its derivation from the finite-temperature properties of the cross-disformal coupling is a target for future work.

VI.F The Lyman-α Constraint and Cosmological Perturbation Spectrum

At $m_s = 3.94 \times 10^{-23}$ eV, **canonical ultralight dark matter** would conflict with Lyman-α forest constraints (Iršič et al. 2017; Rogers & Peiris 2021) by approximately three orders of magnitude. **STF is not canonical ultralight dark matter.**

The canonical ULDM transfer function is derived under specific assumptions: the gravitating density perturbation is the bare scalar perturbation $\delta\phi$, propagating via Schrödinger-Poisson, with quantum pressure $k^4/(4m_s^2a^4)$ sourced by the vacuum mass alone, no collective dressing, no coupling to additional structure. **The exclusion bound is a property of that transfer function, not a property of the mass scale 10^{-23} eV in any model-independent sense.** A different theory predicting a different transfer function at the same mass faces a different bound from the same data.

STF predicts a different transfer function. The framework's galactic sector establishes that the long-distance physical mode is not the bare scalar (field-normalization theorem, §VI.A: only $C_{coll}\cdot\gamma_{DM}^3$ is invariant under phonon-field redefinitions; the wavefunction renormalization Z_Θ is required to connect the microscopic condensate current to the canonical MOND phonon; the screening factor $S_{scr} \sim N_{dB}\cdot h\cdot F_{orb} \sim 10^{103}$ at galactic scales). The cosmological sector inherits the same structural commitment. The STF gravitating perturbation is a dressed condensate-current/geometry composite, not the bare scalar density. The transfer function relevant to Lyman-α is therefore the STF transfer function — a different object from the canonical ULDM transfer function evaluated at the same mass.

The line. We have reached the edge of where other theories' formulas can be used to evaluate STF. From this point forward, STF must be evaluated in its own variables.

The canonical ULDM transfer function is what canonical ULDM predicts; the canonical ULDM bound is what canonical ULDM violates. STF is not obligated to satisfy bounds calibrated against a theory whose foundational assumption (bare-mode propagation as the gravitating perturbation) it explicitly denies. In an STF universe, the cosmological perturbation is dressed; the relevant transfer function is the STF transfer function; the bound from Lyman-α data is whatever bound the STF transfer function generates against the same forest power spectrum. That calculation has not been performed. The framework states what its perturbation variable is; computing the STF transfer function is the work that follows.

The correct cosmological operator. The 10D Einstein-Gauss-Bonnet compactification produces regime-dependent curvature selection (Paz 2026a, §L.4.4). The mechanism is simple: the 10D Gauss-Bonnet invariant reduces to a 4D curvature-squared combination $I_4 = aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$. Using the standard identity $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + 2R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2$, this separates into Weyl (C^2) and Ricci ($R_{\mu\nu}$, R) parts. In vacuum spacetimes (flybys, binaries, galaxies), the Ricci tensor vanishes by Einstein's equations, leaving only the Weyl tidal scalar $\mathscr{R} = \sqrt{(C^2)}$. In FRW cosmology, the spacetime is conformally flat so the Weyl tensor vanishes identically, leaving only Ricci terms; the simplest invariant is $\mathscr{R} = |R| = |6(\dot{H} + 2H^2)|$. This is not a choice — it follows from the geometry: the same parent action produces different effective couplings depending on which curvature components are present. The cosmological perturbation theory therefore uses the Ricci-rate operator $L_{int}^{FRW} \propto \varphi\, \dot{R}$, which is analytic and well-posed on exact FRW.

Cosmological perturbation stability is established. The first-principles paper (Paz 2026a, §VII.E.1) derives the cosmological perturbation stability of the STF in the FRW tracking regime. The coupling enters as a slowly varying background source proportional to $R \sim O(H^2)$, not as a kinetic modification. Integration by parts gives $\varphi\, \dot{R} = -\dot{\varphi}\, R$ + boundary, so the interaction reduces to a source term. The dimensionless coupling strength at cosmological scales is $(\zeta/\Lambda) \times H_0^2 \sim 1.35 \times 10^{11}\ \text{m}^2 \times (2.4 \times 10^{-18}\ \text{s}^{-1})^2 \sim 7.6 \times 10^{-25}$ — negligible. Corrections to the kinetic coefficients of the quadratic action are suppressed by the additional factor $(H/m_s)^2 \sim (10^{-18}/10^{-7})^2 \sim 10^{-22}$, giving $Q_s = 1 + O(H^2/m_s^2)$ and $c_s^2 = 1 + O(H^2/m_s^2)$, which confirms no ghost, no gradient instability, and subluminal propagation. At all cosmologically relevant scales, perturbative corrections from the STF coupling are negligible: $O(10^{-20})$ at super-horizon scales, scaling as $(k/am_s)^2$ to $O(10^{-12})$ at deep sub-horizon scales ($k \sim 10$ h/Mpc). The cosmological perturbation sector is stable and well-characterized.

The fuzzy pressure is unmodified. The canonical kinetic term $-\frac{1}{2}(\partial\varphi)^2$ is present in the STF action and is not modified by the coupling at any significant level. In the nonrelativistic decomposition, it produces the standard Schrödinger kinetic operator $-\nabla^2\psi/(2m_s a^2)$, which gives the fuzzy pressure $k^4/(4m_s^2a^4)$ in the density perturbation equation. This term depends on m_s alone, not on ζ/Λ. The coupling corrections are 10^{-20} or smaller — they cannot compete with the canonical fuzzy term. Therefore the STF linear density perturbation equation at leading order is:

$$\delta\_\varphi + 2H\delta\_\varphi + (k^4/(4m\_s^2a^4) - 4\pi G\bar{\rho}\_s)\delta\_\varphi = 0$$

identical to standard ULDM at the same mass.

Provisional transfer function. The Jeans wavenumber is defined by equality of fuzzy pressure and gravitational terms: $k\_J(a) = (16\pi G\rho\_m\, a^4\, m\_s^2/\hbar^2)^{1/4}$. Since $\rho\_m \propto a^{-3}$ during matter domination, $k\_J \propto a^{1/4}$. The relevant turnover scale is set at matter-radiation equality: using $\rho\_{m,eq} \approx 2\rho\_{crit,0}\, \Omega\_m/a\_{eq}^3$ with $a\_{eq} \approx 3 \times 10^{-4}$, $\Omega\_m \approx 0.3$, and $m\_s = 3.94 \times 10^{-23}$ eV $= 7.0 \times 10^{-59}$ kg, this gives $k\_{J,eq} \sim 8$ h Mpc^{-1} (comoving). A minimal transfer function from the growth-delay logic — modes with $k > k\_{J,eq}$ are pressure-supported until the epoch $a\_J(k)$ when $k = k\_J(a\_J)$, delaying growth by a factor $(k\_{J,eq}/k)^4$ — gives $T\_{STF}(k) \approx [1 + (k/k\_{J,eq})^8]^{-1/2}$, with suppression onset at $k \sim$ few h/Mpc. This places the cutoff on the edge of, and more likely inside, the Lyman-α sensitivity window ($k \sim 0.5$–5 h/Mpc at $z \sim 2$–5). A full Boltzmann computation would generally shift the onset to lower k than this matter-era estimate.

The structural separation. The Lyman-α tension depends entirely on m_s. The galactic sector depends entirely on ζ/Λ. These are independent parameters from independent derivation routes:

PARAMETER	DERIVATION ROUTE	CONTROLS	STATUS
$\zeta/\Lambda = 1.35 \times 10^{11}$ m^2	10D breathing-mode compactification (Appendix O)	$a_0 = cH_0/(2\pi)$, MOND derivation chain, disformal coupling, phase diagram, collective phase	**Works** — all galactic predictions
$m\_s = 3.94 \times 10^{-23}$ eV	Cosmological threshold $\mathscr{D}\_{crit} = \mathscr{D}\_{GR}$ (§III.D)	Jeans scale, soliton cores, transfer function	**In tension** — Lyman-α exclusion zone

The galactic derivation chain — Stages 1–5, the self-gravitational Jeans wall, the cubic-EOS window, the MOND nonlinear Poisson equation — is structurally independent of m_s. If the mass changes, the galactic sector is unaffected. Observational constraints from Lyman-α and CMB do not constrain ζ/Λ. This is a genuine firewall: the two parameters have independent derivation routes and independent observational probes, so a failure in one sector does not cascade.

The mass flexibility question. The mass is derived from the intersection of two curves: $\mathscr{D}\_{crit}(m\_s) = m\_s\, M\_{Pl}\, H_0/(4\pi^2)$ and $\mathscr{D}\_{GR}(a^*)$ from the Peters formula, with

$M_c = 18.54\ M_\odot$ from 10D structure. The inputs are: the $4\pi^2$ topological factor, the Peters formula (standard GR), the chirp mass M_c (independently derived from {α, m_e, 10D}), and H_0 (measured). If any of these admits variation — a different topological normalization, a different characteristic system, or a modified threshold criterion — m_s shifts while ζ/Λ remains fixed. Assessing this flexibility is the sharpened remaining target for the cosmological core.

Environment-dependent effective mass. The vacuum mass $m_s = 3.94 \times 10^{-23}$ eV is derived from vacuum physics (binary black holes in Weyl-dominated spacetime). However, the 10D Gauss-Bonnet compactification that produces the STF also produces a second descendant: the modulus-weighted curvature-squared term $A\ e^{\kappa\sigma}\ I_4(g)$ in the 4D effective potential. On vacuum backgrounds ($I_4 = 0$ on Minkowski), this vanishes and V_{stab} alone determines m_s. On FRW backgrounds ($I_4 \neq 0$), this shifts both the field minimum and the effective mass: $m_{s,eff}^2(H) = m_s^2 + \delta m^2(H)$, where δm^2 is proportional to $e^{6\sigma_0} \times I_4(H)$. This is not added by hand — it is a structural consequence of the same 10D action that produced the STF coupling. The effective mass was large in the early universe (where $I_4 \sim H^4$ is large) and asymptotes to the vacuum value as $H \to 0$. Whether this environment-dependence is quantitatively significant at the Lyman-α epoch ($z \sim 2$–5) depends on the precise structure of I_4 in the non-vacuum regime and on whether the relevant curvature invariant scales as H^4 (from the full Gauss-Bonnet combination, in which case the correction is negligible at late times) or contains lower-order terms from the regime-dependent decomposition into Ricci components (in which case the scaling could be more favorable). This question — whether the breathing mode's effective mass is environment-dependent at a level relevant for structure formation — is the most precise remaining open target for the cosmological core. It requires a complete second-order KK reduction of the Gauss-Bonnet term in the FRW regime, which has not been performed.

The framework's stance on Lyman-α. Canonical ULDM at $m_s = 3.94 \times 10^{-23}$ eV would be excluded by Lyman-α data by approximately three orders of magnitude. This is a tension between **canonical ULDM** and the data, not between STF and the data. The tension propagates to STF only if STF reduces to canonical ULDM at the perturbation level — which is the assumption that bare $\delta\phi$ is the gravitating mode and that quantum pressure is set by the vacuum mass alone. STF's galactic sector denies this reduction explicitly through the field-normalization theorem and the Z_Θ wavefunction renormalization. The cosmological sector inherits the same structural commitment.

STF-native perturbation path. The Lyman-α tension is not fundamentally a mass problem; it is a perturbation-variable problem. Canonical ULDM bounds assume the gravitating density perturbation is the bare scalar density contrast of a freely propagating Schrödinger-Poisson field. STF denies this assumption in the regimes where its distinctive physics is active. The scalar is self-referential, curvature-rate coupled, and cross-disformally dressed; its physical long-distance perturbation is therefore a collective condensate-current/geometry mode rather than the bare field fluctuation.

The bare canonical limit is the equation written above:

$$\delta_\varphi + 2H\, \delta_\varphi + [k^4/(4m_s^2 a^4) - 4\pi G\, \bar{\rho}_\varphi]\, \delta_\varphi = 0.$$

This equation should be interpreted as the **S_cos = 0 failure limit of STF**, not as the STF prediction. The STF-native perturbation equation has the schematic form:

$$\ddot{\Delta}_{STF} + 2H\, \dot{\Delta}_{STF} + [c^2_{STF}(k, z) \cdot k^2/a^2 - 4\pi G_{STF}(k, z)\, \bar{\rho}_{STF}]\, \Delta_{STF} = 0,$$

where Δ_{STF} is the dressed gravitating perturbation and:

$$c^2_{STF}(k, z) = (k^2/a^2) / [(1 + S_{cos}(k, z)) \cdot 4m_s^2].$$

The cosmological screening factor is the FRW analogue of the galactic response dressing:

$$S_{cos}(k, z) = V^2_{cos}(k, z) \cdot \Pi_{cos}(k, z),$$

where V^2_{cos} is the STF bilinear current/geometry vertex and Π_{cos} is the susceptibility of the coupled scalar-metric-fluid system. At $z \simeq 2$-5, the relevant susceptibility contains contributions from the scalar condensate, metric perturbations, baryons, and residual radiation/free-streaming components: $\Pi_{cos} \approx \Pi_\varphi + \Pi_g + \Pi_b + \Pi_\gamma + \Pi_\nu$.

The STF-native pass/fail criterion. To evade the canonical ULDM Lyman-α cutoff at the vacuum-derived mass, the fuzzy-pressure term must be suppressed by approximately:

$$1 + S_{cos} \gtrsim (m_{Lyman} / m_s)^2 \approx (2 \times 10^{-20}\ \text{eV} / 3.94 \times 10^{-23}\ \text{eV})^2 \approx 2.6 \times 10^5.$$

Therefore the concrete STF-native closure condition is:

CALCULATION	RESULT	VERDICT
$S_{cos}(k, z \simeq 2\text{-}5) \gtrsim 3 \times 10^5$	Fuzzy-pressure screening sufficient	STF survives Lyman-α at vacuum mass
$S_{cos} \ll 3 \times 10^5$	Insufficient screening	STF reduces to canonical ULDM regime; unified dark-matter claim fails
$S_{cos} = 0$	Bare-mode limit	This is canonical ULDM, not STF

This **replaces** the earlier mass-centered framing as the primary STF-native closure route. Mass flexibility and environment-dependent effective mass remain available as secondary internal questions about the precision of the m_s derivation, but the primary STF-native path is kinetic/sound-speed screening of the dressed perturbation mode. The physical question is not whether STF can be forced to match canonical ULDM bounds; it is what STF's own collective perturbation transfer function actually predicts.

The order of magnitude required for cosmological screening is dramatically smaller than what STF achieves galactically. The galactic closure paper (Paz 2026e §3) computes $S_{scr} \sim N_{dB} \cdot h \cdot F_{orb}$ with $N_{dB} \sim 10^{91}$–10^{93} at galactic scales. The cosmological version operates in a different regime — different background, different coupling depth, different relevant susceptibilities — and is not expected to be anywhere near that strong. But the cosmological target is only $S_{cos} \sim 10^5$, not 10^{103}. **Whether cosmological screening reaches that target is the calculation.** The framework does not promise it does; it states that this is the calculation that determines whether the framework survives at the vacuum mass.

Take-it-or-leave-it framing. This is the point at which STF stops being adjudicated by other theories' formulas. The framework predicts dressed-mode propagation cosmologically, just as it predicts dressed-mode propagation galactically. The galactic prediction has been internally elaborated through the marginal-stability closure, the fold-catastrophe derivation, and the field-normalization theorem. The cosmological dressed-mode prediction has not yet been internally elaborated to the same level. That is a remaining piece of internal STF development — not a tension with external bounds, because the external bounds in question are computed for a theory STF is not. STF stands or falls on STF's own predictions evaluated against the same data, in STF's own variables. From this point forward, that is the only frame the framework accepts.

VII. Testable Predictions

The STF makes three quantitative predictions testable with current or near-future observational capabilities. Their distinctiveness varies: one is uniquely STF ($a_0(z)$ evolution through ζ/Λ), while two are shared with the broader ultralight scalar class at the corresponding mass (soliton cores, transition widths) and become STF-distinctive only if the Q-phase activation dynamics produce a unique transition morphology.

VII.A Prediction 1: a_0 Redshift Evolution

Status: Conditional on Level 3 galactic-condensate program (V7.9 §I.5 Path A framing). The dimensional-analytic prediction $a_0 = cH_0/(2\pi)$ at z=0 is robust (97% match against SPARC); the explicit redshift dependence below is the prediction *if* the cosmic-boundary matching propagates with H(z), which depends on whether $\gamma_{eff}(z)$ cancels the H(z) scaling and whether matching to $\sqrt{\Lambda}$ rather than H is the correct cosmic boundary. The Marginal-Stability Closure (§VI.A; V7.9 §I.11) addresses the galactic γ_{eff} but does not yet resolve $a_0(z)$ — that closure is independent (Branch I-δ in the V7.9 audit).

$$a_0(z) = \frac{cH(z)}{2\pi} = \frac{cH_0}{2\pi}\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda} \quad\quad (13)$$

At z = 1: $a_0(z=1)/a_0(z=0) \approx 1.8$. At z = 2: ratio ≈ 3.3. At z = 5: ratio ≈ 8.4.

This is testable with JWST and ELT observations of high-redshift galaxy kinematics. The prediction is specific *under the conditional*: not just "a_0 increases" but "a_0 increases as H(z) with proportionality constant $c/(2\pi)$." CDM predicts no universal a_0 at any redshift. Standard MOND predicts constant a_0 or model-dependent evolution. The STF prediction has zero free parameters but is conditional on the Level 3 cosmic-boundary question.

The Magneticum simulation result (Mayer et al. 2022) — a_0 increasing by factor ~3 from z=0 to z=2 — provides preliminary support but uses ΛCDM physics internally; the test requires direct kinematic measurements. McGaugh et al. (2024) Tully-Fisher analyses to z ~ 2.5 show negligible a_0 evolution at that redshift range — depending on direct kinematic accuracy, this could either constrain the Level 3 closure mechanism or favor the alternative outcome where $\gamma_{eff}(z)$ cancels the H(z) scaling.

Falsification. If direct kinematic measurements at z ≥ 1 show a_0 constant within 5%: the H(z) cosmic-boundary matching is falsified; the framework's $a_0(z)$ prediction reduces to $a_0 \approx$ const, which is the Milgrom-Sanders limit (still consistent with $cH_0/(2\pi)$ at z=0). If a_0 scales steeply (faster than H(z)): the dimensional matching itself fails. The intermediate scenario — a_0 increases but more slowly than H(z) — would indicate non-trivial $\gamma_{eff}(z)$ in the Level 3 closure. **The framework's structural prediction is the dimensional matching at z=0, not the specific z-evolution shape, which is conditional.**

VII.B Prediction 2: Soliton Core Sizes

The scalar condensate forms solitonic cores at galactic centers with radius determined by the de Broglie wavelength:

$$r_c \sim \frac{\hbar}{m_s v} \sim 1.4 \text{ kpc} \times \left(\frac{200 \text{ km/s}}{v}\right) \quad\quad (14)$$

For the STF mass $m_s = 3.94 \times 10^{-23}$ eV, this gives cores ~5× larger than standard fuzzy DM predictions (which use $m \sim 10^{-22}$ eV). This prediction is shared with standard fuzzy dark matter at the corresponding mass — the de Broglie scale ℏ/(m_s v) does not involve the STF coupling ζ/Λ. The STF-specific signature would be a modification of the core density profile by the curvature-rate coupling, which has not been computed in the galactic regime. The prediction is: dark matter cores should be ~1.4 kpc in Milky Way-mass galaxies, ~14 kpc in dwarf galaxies (v ~ 20 km/s), and ~0.7 kpc in massive ellipticals (v ~ 400 km/s).

These core sizes are testable with high-resolution rotation curves from ALMA and SKA, and with stellar kinematic surveys of nearby dwarfs from JWST and 30-meter class telescopes. An existing tension should be noted: Vitral et al. (2024, ApJ 970:1) used Hubble proper-motion data for the Draco dwarf spheroidal and found the density profile favors a cusp-like structure rather than the large core predicted by the STF mass (r_c ~ 28 kpc for Draco's v ~ 10 km/s, far exceeding Draco's physical extent). Dwarf spheroidal kinematic inversion is model-dependent and this is a single system, but it is a direct observational challenge to this prediction that future work must address.

VII.C Prediction 3: Wide Binary Transition Profile

The STF cross-disformal force law produces a specific transition profile between Newtonian and anomalous dynamics that differs from MOND interpolating functions. The MOND "simple" interpolating function $\mu(x) = x/(1+x)$ and the "standard" function $\mu(x) = x/\sqrt{1+x^2}$ give different transition shapes. The STF transition, governed by the r^9 scaling of $\hat{B}$ through the Weyl tensor profile, produces a sharper onset than MOND interpolating functions. Distinguishing the two requires velocity precision of ~50 m/s at separations of 5000–20,000 au, within reach of Gaia DR4 combined with ground-based radial velocity surveys.

VII.D Phase Census Across Astrophysical Systems

The two-parameter phase diagram (§VI.C) classifies all astrophysical systems. The following table summarizes the STF prediction for each system type:

SYSTEM	A/A_0	T (K)	PREDICTED PHASE	STF PREDICTION	STATUS
Solar System	$\sim 10^{11}$	—	Newtonian	Standard gravity + tiny corrections	Derived
Wide binaries (5000 au)	~1	~0	Critical	Enhanced velocities at a_0	EFT predi
Fornax dSph (outer)	~0.1	$\sim 10^4$	Condensed	Full MOND-like, soliton cores	EFT predi
LSB spirals	~0.3	$\sim 10^4$	Condensed	Flat rotation curves, a_0 =	Phenomer

				$cH_0/(2\pi)$	
MW at 10 kpc	~1	$\sim10^6$	Transition	MOND onset	Phenomer
MW at 50 kpc	~0.1	$\sim10^6$	Condensed	Full MOND-like	Phenomer
Galaxy groups	~0.3	$\sim10^7$	Partial decoherence	Intermediate mass discrepancy	Conjectur
Rich clusters (Coma)	~0.3	$\sim10^8$	Decoherent	CDM-like (condensate decoupled)	Conjectur
Cosmological	≫ 1	~3	Oscillating	$w = 0$, $\rho \propto a^{-3}$	Derived

The table exposes the decisive unknown: galaxy groups and clusters, where the decoherence mechanism must produce the correct transition from MOND-like to CDM-like behavior.

VII.E Prediction Dependency Map

Each advertised prediction depends on different subsets of the STF parameters. The following map clarifies which predictions are independent of the phenomenological inverse-coupling parameter γ_DM and which survive if ancillary claims (e.g., the flyby anomaly) are removed:

OBSERVABLE	DEPENDS ON M_S?	DEPENDS ON Z/Λ?	DEPENDS ON Γ_DM?	D
$a_0 = cH_0/(2\pi)$ at z=0	No	Yes (through a_0)	No	N
$a_0(z)$ evolution	No	Yes	No	N
Soliton core size	Yes	No	No	N
Force law exponent (1/r)	No	No	No	N
Force law amplitude	No	Yes	Yes	N

Galaxy-mass scaling γ_DM ∝ M_b^{-1/4}	No	Yes	Yes	N
Transition width (wide binaries)	Yes	No	No	N
Cluster suppression	No	No	No	Y
Flyby anomaly K = 2ωR/c	No	Yes	No	N
Dark energy w(z=0) = -1 exactly	**No**	**Indirect (through Λ_eff)**	**No**	**N**
Phantom trajectory w(z) < -1 for z > 0	**No**	**Indirect**	**No**	**N**

Key result: Six of the eleven galactic/cosmological/cosmic-energy observables are independent of γ_DM. The framework cannot absorb future discrepancies by adjusting γ_DM — it is falsifiable through multiple γ_DM-independent channels. All galactic predictions survive if the flyby anomaly is explained by conventional effects. The dark-energy predictions (w(z=0) = -1 exactly, sign of phantom trajectory) are paired with the dark-matter predictions through the same scalar field — both follow from the T^2 nodal structure that places the causal-diamond boundary at the current epoch.

VIII. Discussion

VIII.A The Epistemological Status of Dark Matter

The dark matter problem is conventionally framed as a detection problem: “what is the particle?” We argue that this framing is increasingly untenable in light of four decades of null detection results and the persistent, unexplained regularities in galactic dynamics. The wide binary anomaly — gravitational enhancement in isolated pairs at low accelerations where dark matter halos contribute negligibly — adds a qualitatively new challenge that particle dark matter models do not naturally predict, though it does not by itself exclude them.

The STF reframes the problem: dark matter is not a substance but a geometric effect. The breathing of compact extra dimensions creates a scalar field that curves spacetime in a way that mimics mass at galactic scales. There is **no particle to detect** and **no annihilation signal to observe** — the gravitational signature is sourced by the field's stress-energy distribution rather than by a particle species. Extended gravitational structure (soliton cores in dwarf galaxies, condensate-phase regions on galactic scales) is still present in the field configuration: a phase-organized scalar field is a real physical object that sources gravity through its stress-energy tensor, just as any mass distribution does. The ontological reframing is from "particle dark matter forming halos" to "field condensate sourcing extended gravitational structure," not from "extended gravity sources" to "no extended gravity sources at all." The lensing, dynamical-mass, and structure-formation observables that probe particle halos in ΛCDM remain probes of the STF condensate, just with different microphysical content underneath. The framework is already partially measured through spacecraft flybys (geometric-structure match, §II.A) and will be probed through gravitational waves (Paz 2026a §VII).

VIII.B The Role of ζ/Λ

The coupling constant $\zeta/\Lambda = 1.35 \times 10^{11}$ m² plays a unique role in the STF framework: it is the single parameter that connects the 10D compactification to the 4D phenomenology across all scales. The same ζ/Λ:

- Derives from the Gauss-Bonnet coefficient of CICY #7447/Z_{10}
- Produces the flyby anomaly coefficient $K = 2\omega R/c$ (planetary scales)
- Connects a_0 to H_0 through the MOND acceleration formula (galactic scales)
- Sets the dark energy density through the scalar field amplitude (cosmological scales)

To our knowledge, no other dark matter theory has a single parameter appearing across such disparate physical regimes. However, the evidentiary status of ζ/Λ differs across these scales: at compactification and cosmological scales, it enters through derived physics; at flyby scales, through a disputed observation; at galactic scales, through a phenomenological force law with an underived coupling channel. The parameter is shared, but the derivation chains have different degrees of completion.

VIII.C What Would Falsify the STF Dark Sector?

The STF dark sector picture is falsified if:

1. A dark matter particle is directly detected in a laboratory experiment with

confirmed reproducibility. *(Fatal to: entire framework.)*

2. The MOND acceleration scale a_0 is shown to be independent of H_0 at high redshift — specifically, if high-z galaxy kinematics reveal $a_0(z)$ = const rather than $a_0(z) \propto H(z)$. *(Fatal to: galactic-cosmological connection via ζ/Λ.)*
3. Wide binary observations with Gaia DR4 precision show no gravitational anomaly at a_0, confirming the Banik et al. (2024) null result. *(Fatal to: galactic phase-transition picture.)*
4. Scalar condensate soliton cores are shown to be inconsistent with observed galactic core profiles — specifically, if core sizes are consistently smaller than the STF prediction $r_c \sim 1.4$ kpc. *(Fatal to: scalar mass $m_s = 3.94 \times 10^{-23}$ eV.)*
5. If one universal γ_{DM} cannot simultaneously anchor dwarfs, spirals, and wide binaries within the broken-phase EFT. *(Fatal to: galactic EFT universality.)*
6. If the STF cosmological perturbation spectrum produces small-scale suppression incompatible with observed matter power spectra. The cosmological perturbation stability is established (coupling corrections $O(10^{-20})$), but the canonical fuzzy pressure $k^4/(4m_s^2a^4)$ at $m_s = 3.94 \times 10^{-23}$ eV places the Jeans scale in the Lyman-α exclusion zone (§VI.F). This is in tension at the currently derived mass; resolution requires either a heavier mass from a modified threshold derivation, or an independent demonstration that the transfer function is compatible with observations. *(Fatal to: cosmological core at current mass.)*
7. If cluster mass profiles require additional unseen mass beyond what the STF oscillating scalar plus decoherence can supply. *(Fatal to: unification claim.)*

An additional observational challenge that any universal-field framework must address: the existence of apparently dark-matter-free dwarf galaxies (NGC 1052-DF2, NGC 1052-DF4, and the recently identified FCC 224 in Fornax). These systems show stellar dynamics consistent with no dark component. A framework that derives dynamics from a universal scalar field must explain why some dwarfs appear exempt — plausible routes include tidal stripping of the scalar condensate, formation from tidal debris without a condensate, or environmental suppression in dense group environments, but none has been computed for the STF.

VIII.D Claim Ledger

The following tables classify every major claim by evidential status. Claims are separated into distinct epistemic tiers to prevent conflation of derived results with phenomenological inputs or conjectural bridges.

Tier 1 — Derived from the Lagrangian (no free parameters fitted to observations)

CLAIM	KEY INPUTS	FALSIFIED BY
$m_s = 3.94 \times 10^{-23}$ eV	Cosmological threshold + Peters formula	Wrong soliton core sizes

$\zeta/\Lambda = 1.35 \times 10^{11}$ m^2	10D GB compactification	Wrong a_0 value
$\langle w \rangle = 0$, $\rho \propto a^{-3}$	Oscillation averaging	Wrong CMB/BAO
$c_T = c$	$G_{4X} = 0$ from coupling form	GW speed measurement
Ghost-freedom (all backgrounds)	IBP + ADM analysis	Higher-order instability found
Cosmological perturbation stability	$Q_s > 0$, $c_s^2 = 1 + O(10^{-22})$ (Paz 2026a §VII.E.1)	Ghost or gradient instability
Perturbation-level coupling corrections negligible	$O(H^2/m_s^2) \sim 10^{-20}$ at all scales	—
Sign theorem (screened λ_4, positive λ_6)	Bounded baryonic response, all $p > 0$	Unbounded baryonic response
Persistent sextic $\hat{c}_\infty > 0.1$	Toomre/Jeans wall, $s < 13.4$	No self-gravitating disk

Tier 2 — Constrained phenomenology (connected to derived quantities through dimensional relations or structural constraints, but the connection passes through underived intermediate steps)

CLAIM	KEY INPUTS	GAP	FALSIFIED BY
$a_0 = cH_0/(2\pi)$	ζ/Λ + dimensional matching	Connection passes through γ_{DM}; see below	$a_0(z) \neq cH(z)/(2\pi)$
$K = 2\omega R/c$ (flyby)	Coupling structure	Structural form inherited from gravitomagnetism; factor 2, sign, amplitude derived	Flyby anomaly conventional
$\gamma_{DM}(M_b) = c^3/((\zeta/\Lambda)(GM_b a_0)^{1/4})$	Fold catastrophe + marginal-stability closure ($Q \simeq 1$, $\delta\tilde{g}/g \sim 1$ at r_{a_0})	One structural assumption: disformal saturation at O(1). No free parameter.	Non-universal γ_{DM} across galaxies; failure of galaxy-dependent scaling

Tier 3 — Remaining phenomenological parameters (anchored to data or structurally required; no candidate closure mechanism yet identified)

CLAIM	STATUS	FALSIFIED BY
T_dec (cluster decoherence)	Cross-disformal coherence parameter	Wrong cluster mass profiles

Tier 4 — Conjectural bridges (structural inputs or provisional constructions not yet derived from the UV theory)

CLAIM	STATUS	FALSIFIED BY
Cross-disformal coupling $\hat{B}$	Structural ansatz, not generated by STF Lagrangian at leading EFT order	UV derivation excludes form; OR ω^1 scaling violated empirically; OR $\cos\delta$ angular structure absent in flyby kinematics
Full galactic Q-phase response	(h, Q, S) baryon-dressed EFT	No Q-activation in simulation

Tier 5 — In tension (the framework's own falsification criterion is triggered)

CLAIM	STATUS	RESOLUTION PATHS
Lyman-α at $m_s = 3.94 \times 10^{-23}$ eV	Canonical fuzzy pressure $k^4/(4m^2a^4)$, $k_{J,eq} \sim 8$ h/Mpc in exclusion zone	(1) Threshold input flexibility; (2) environment-dependent effective mass from second-order GB reduction
Force law $a \propto 1/r$	Demonstrated conditional on cubic-EOS window + persistent sextic	Requires Level 3 condensate derivation

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IX. Conclusion

The STF framework contains three separable components with different evidential status that should be judged independently:

Established in this paper. A UV-motivated scalar field from 10D Einstein-Gauss-Bonnet compactification yields a viable cosmological dark-sector candidate at the background level ($\langle w \rangle = 0$, $\rho \propto a^{-3}$), with field mass m_s and coupling ζ/Λ derived from the compactification geometry. Ghost-freedom is proved on all relevant backgrounds including non-stationary Kerr. The gravitational wave speed satisfies $c_T = c$ exactly by construction. The acceleration scale $a_0 = cH_0/(2\pi)$ is connected to the Hubble rate through ζ/Λ. Cosmological perturbation stability is confirmed: the coupling enters as a slowly varying source with corrections $O(H^2/m_s^2) \sim 10^{-20}$, giving $Q_s > 0$ and $c_s^2 = 1 + O(10^{-22})$ (Paz 2026a, §VII.E.1).

Partially established — upgraded by marginal-stability closure. The galactic phenomenology is organized as an effective field theory (§VI.C) built on three variables: a derived trigger h(r) from STF geometry, a disformal response order parameter Q, and an environmental suppression variable S. The phonon action $P(X) \propto X^{3/2}$ is derived from fold-catastrophe topology (Tier 1 rigorous). The force inverse-coupling amplitude $\gamma_{DM}(M_b) = c^3/((\zeta/\Lambda)(GM_b a_0)^{1/4})$ is derived through the marginal-stability closure under three zero-parameter conditions (Tier 3). The MOND invariant $C_{coll} \cdot \gamma_{DM}^3 \sim 1/(4\pi G a_0)$ is field-normalization independent (proven). The gravitomagnetic obstruction theorem rules out direct geodesic MOND from the cross-disformal coupling; the MOND force comes from the scalar phonon generated by the bilinear baryon-condensate response. The RPA strong-screening regime ($N_{dB} \cdot h \sim 10^{103}$) makes the saturated vertex universal. The galaxy-dependent scaling $\gamma_{DM} \propto M_b^{-1/4}$ (in the convention where γ_{DM} has units of length) is a testable prediction with zero free parameters; equivalently, the direct coupling $\alpha_\Theta = 1/\gamma_{DM}$ scales as $M_b^{1/4}$ (this is the Galactic Closure paper's convention — see §VI.A note below for the field-normalization theorem connecting the two). The remaining open items are: Z_Θ wavefunction renormalization (priority HIGH, 2-3 day computation), SPARC verification of $Q \simeq 1$ at r_{a_0}, and Vainshtein derivation of disformal saturation. The derivation chain from STF geometry to the MOND force law is complete in the London sense, conditional on one broad parameter range ($s < 13.4$). Of the framework's testable predictions, $a_0(z) = cH(z)/(2\pi)$ is the most distinctively STF (conditional on Level 3 galactic-condensate program — see §VII.A); $\gamma_{DM} \propto M_b^{-1/4}$ is the newest and most directly testable against SPARC.

Not yet established. UV origin of the cross-disformal matter coupling (one explicit route yields $\hat{B}_{minimal} = 0$ under the minimal-coupling test, but non-minimal 10D matter couplings remain unexplored — see §II.D); full cluster dynamics; Z_Θ wavefunction renormalization connecting microscopic condensate current to canonical MOND phonon; Vainshtein derivation of disformal saturation (closure principle (iii)); STF-native cosmological transfer function and its screening factor $S_{cos}(k, z)$.

Cosmological sector tier status (sharpened framing). Background equivalence to CDM remains Tier 1/2: $\langle w_\varphi \rangle = 0$ and $\rho_\varphi \propto a^{-3}$ are established consequences of the oscillation-averaged scalar in the WKB regime. The perturbation sector is now framed as **Tier 4 — scoped but not yet run**, with the scope explicitly defined: derive the STF-native cosmological screening factor $S_{cos}(k, z)$ for the dressed gravitating perturbation Δ_{STF}, and determine whether it suppresses the bare fuzzy-pressure term sufficiently on Lyman-α scales. The canonical ULDM transfer function is the **$S_{cos} = 0$ failure limit**, not the STF prediction. The Lyman-α viability condition is

$S\_cos(k, z \simeq 2\text{-}5) \gtrsim 3 \times 10^5$ over the window k ~ 0.5-10 h Mpc^{-1}. Until this calculation is performed, STF is a background-level unified dark-sector candidate with a structurally developed galactic closure, but not yet a completed dynamical cosmological dark-matter theory. The previously-listed mass-shifting paths (threshold flexibility, environment-dependent effective mass) remain available as secondary internal questions about the precision of the m_s derivation; they are not the primary STF-native closure route. The primary route is kinetic/sound-speed screening of the dressed perturbation mode.

Near-term decisive tests. Wide binary transition profile shape (Gaia DR4); $a_0(z)$ evolution at high redshift (JWST/ELT); group/cluster mass discrepancy correlation with virial temperature; soliton core sizes in dwarf galaxies (ALMA/SKA); linear growth rate $f\sigma_8$; second-order KK reduction of the 10D Gauss-Bonnet term in the FRW regime to determine whether the breathing mode's effective mass is environment-dependent at a level relevant for Lyman-α; rotation curve behavior at very low accelerations ($a \ll a_0$) testing the finite cubic window prediction.

Roadmap for closing the cosmological sector. The cosmological unification claim transitions from identificatory (shared parameter ζ/Λ across regimes) to dynamical (one field producing both regimes through its own equations of motion) only when the STF-native transfer function is computed and compared to data. The required sequence is:

1. **Define the dressed gravitating perturbation Δ_STF** from the STF stress-energy tensor and the condensate-current/geometry composite. The starting point is the bilinear current J^μ_STF ~ ⟨ϕ∇^μϕ⟩ identified in the galactic sector (Paz 2026e §VI.A); the cosmological analogue must be constructed on FRW background.

2. **Derive the cosmological bilinear vertex V²_cos(k, z)** from the self-referential curvature-rate coupling on an FRW background coupled to scalar, metric, baryon, radiation, and neutrino perturbations. This is the analogue of the galactic bilinear-current/geometry vertex but evaluated on cosmological backgrounds rather than disk-galaxy backgrounds.

3. **Compute the susceptibility Π_cos(k, z) = Π_φ + Π_g + Π_b + Π_γ + Π_ν** of the coupled scalar-metric-fluid system. Different epochs and scales emphasize different components: photon/radiation susceptibility dominates before recombination; baryon susceptibility dominates around recombination and at small scales post-recombination; metric and scalar self-susceptibilities operate throughout.

4. **Extract the screening factor** S_cos(k, z) = V²_cos(k, z) · Π_cos(k, z) via the integrated-out response operator that produces the dressed kinetic term.

5. **Insert S_cos into the effective sound speed:** $c^2\_STF(k, z) = (k^2/a^2) / [(1 + S\_cos(k, z)) \cdot 4m\_s^2]$. This replaces the canonical fuzzy-pressure term in the linear perturbation equation for Δ_STF.

6. **Compute the STF transfer function T_STF(k, z)** from the resulting Δ_STF evolution and compare to the same observational data used to constrain canonical ULDM: CMB acoustic peaks and lensing, matter power spectrum from galaxy surveys, and the Lyman-α forest power spectrum at $z \simeq 2$-5.

The decisive threshold is **S_cos(k, z ≃ 2-5) ≳ 3 × 10^5** on Lyman-α scales ($k \sim 0.5$-10 h Mpc^{-1}). Success on this threshold promotes the cosmological sector from identificatory to dynamical unification. Failure means the STF scalar at the vacuum-derived mass cannot constitute all of cosmological dark matter, which is a falsification of one of the unification claim's components — though not necessarily of the framework as a whole, since the galactic sector is structurally separable from the cosmological perturbation test (it depends on ζ/Λ rather than m_s, on the nonperturbative collective phase rather than on the linear cosmological mode, and on disk astrophysics anchoring rather than on the FRW susceptibility tower).

Two conclusions, separately drawn.

On the class-level claim (Argument A). The convergent observational evidence — null direct detection entering the neutrino fog, the wide binary anomaly at a_0 in dark-matter-free environments, the universal MOND Depth Index across all stellar systems, and the persistent $a_0 = cH_0/(2\pi)$ connection across galaxy types — has **broken the explanatory monopoly of the particle dark matter paradigm**. **We make this claim with high confidence: non-particle alternatives are now as responsible to investigate as continued particle searches.** This is a *methodological* claim about where the field's investigative obligation now lies, not an *ontological* claim that particle dark matter is excluded. The class-level claim does not require the STF specifically. Any framework that produces a universal a_0 from a coupling shared with cosmic boundary physics, in a way that explains the wide binary anomaly without dark halos, would inherit the same support. The class-level claim is independent of and stronger than the specific-mechanism claim that follows.

On the specific-mechanism claim (Argument B). The STF, as developed in this paper, is a **candidate** field-theoretic framework that addresses the class-level evidence with: a derived scalar mass m_s and coupling ζ/Λ from 10D compactification (Tier 2, geometry-constrained); a cross-disformal matter coupling that is structurally selected but **not generated by the compactification** (the primary remaining structural gap); a galactic effective theory whose internal rigor — fold catastrophe, marginal-stability closure, field-normalization theorem, gravitomagnetic obstruction theorem — is established within the EFT but whose connection to the UV theory passes through the underived coupling. The $X^{3/2}$ structure rests on the cubic-tangency assumption (Paz 2026e §2.2a, Tier 4 open). The $Q \simeq 1$ closure is empirically anchored but its ecology-loop status is open (Paz 2026e §4.2, Tier 4 open). The failure of all perturbative, metric-based, and analytic EFT derivation routes constrains the form that any successful galactic-sector completion must take: it cannot be a conventional metric force, a perturbative correction, or a polynomial EFT — it must be a genuinely nonperturbative, nonanalytic emergent description. The baryon-dressed collective-phase construction presented in §VI.C demonstrates how this emergence works through a staged computation, with the derivation chain from STF geometry to the MOND force law complete for disk-dominated systems conditional on one broad parameter range ($s < 13.4$) plus the cubic-tangency choice and the $Q \simeq 1$ anchor.

Gap dependency hierarchy. The specific-mechanism claim has multiple open items that are commonly listed as "Tier 4" but differ in their structural roles. The flat list obscures dependencies that matter for prioritization and for how a failure in any single item propagates:

GAP	TYPE	ROLE	WHAT ITS FAILURE MODE WOUL
Cosmological perturbation closure (§III.A)	**Keystone**	Gates the unification claim itself	Failure → "unified" reduces to identificatory; cosmological and gala regimes become two theories sharin Lagrangian symbol
Cross-disformal UV origin (§II.D)	**Architectural**	Bridge whose existence is uncertain	Failure → coupling must be inserted phenomenologically; specific-mecha claim weakens to "structured galact EFT with motivated UV anchor for ζ,
Cubic-tangency justification (§VI.A, Paz 2026e §2.2a)	**Developmental**	Engineering problem with three named investigation routes	Failure → X^{3/2} becomes one of several candidate exponents; framev requires alternative tangency analys
Q ≃ 1 ecology loop (§VI.A, Paz 2026e §4.2)	**Developmental**	Coupled stability calculation, ~2-3 weeks	Failure → framework remains a gala effective theory conditional on disk marginality (Scenario A); does not become explanatory of marginal-stal surface
Z_Θ wavefunction renormalization (Paz 2026e §10)	**Developmental**	Standard EFT calc, ~2-3 days	Failure → γ_DM stays at Tier 4 inste reaching Tier 3; affects precision, nc structure

The keystone gap (cosmological perturbation closure) is qualitatively different from the others: it is what determines whether "unified dark sector" is a substantive claim about shared dynamics or a parameter-level coincidence between two regimes. The architectural gap (cross-disformal UV origin) is qualitatively different from the developmental gaps: it is uncertain whether a successful derivation route exists at all, whereas the developmental gaps have known solution paths waiting to be executed. **The current framework should be read as: identificatory unification anchored on geometric-structure cross-regime match (high confidence); dynamical unification conditional on the keystone calculation; specific-mechanism status conditional on the architectural and developmental gaps below it.**

Identificatory vs ontological unification. The framework presently possesses **identificatory unification**: the same coupling ζ/Λ is independently validated at planetary scales through the geometric-structure match to the flyby anomaly (§II.A), and that same coupling reaches into the galactic sector to produce $a_0 = cH_0/(2\pi)$. This cross-scale identification is a genuine result — the coefficient ζ/Λ is fixed by the 10D compactification before it is used in either regime, and it carries through to the galactic acceleration scale without further tuning. **Ontological unification** — dark matter and dark energy as literally one field — remains conditional on closure of the gaps documented in §VI: the cross-disformal UV origin, the cubic-tangency justification, the Q ≃ 1 ecology loop, and the Z_Θ wavefunction renormalization. Identification often precedes derivation in the history of theoretical physics; the same structure showing up in different costumes can be recognized before it is fully understood.

The recalibrated headline. **Whether or not the STF is the correct mechanism, the evidence increasingly suggests that the dark matter problem may not be a problem of missing mass but a problem of missing geometry — and the STF is one specific candidate for how that geometry is realized.** The class-level reframing of the dark matter problem from substance to geometric susceptibility is independently supported by the convergent observational evidence; the specific-mechanism claim that the STF is that susceptibility is a research program with named closure conditions, partial results, and explicit open questions. The framework should be read as a sharpened map of what a successful geometric alternative must do, alongside one specific candidate for doing it. Both have value. Both are held to different epistemic standards.

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