The Cross-Disformal Matter Coupling as the Unique Mechanism for the Anderson Flyby Anomaly in Scalar-Tensor Gravity

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Abstract

The Selective Transient Field (STF) derives the Anderson flyby formula $K = 2\omega R/c$ from the Lagrangian coupling $\varphi(n^\mu\nabla_\mu\mathcal{R})$, matching the empirical coefficient to 99.99% with zero free parameters [Paz 2026c]. This paper establishes the matter coupling mechanism that produces the flyby amplitude.

We show that the cross-disformal matter metric $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \hat{B}(\partial_\mu\varphi\,\partial_\nu\mathcal{R} + \partial_\mu\mathcal{R}\,\partial_\nu\varphi)$ *is the* ***unique*** *scalar-tensor coupling satisfying all structural requirements of the Anderson formula: linear scaling in planetary rotation rate (ω^1), linear scaling in asymptotic velocity* (V_∞^1), convergent trajectory integral ($1/r^{10}$), declination-dependent angular structure ($\cos\delta$), and non-zero work ($F\cdot v \neq 0$). Six candidate couplings are tested; five are eliminated by these requirements.

The cross-disformal structure is an **additional structural input** — not generated by the STF Lagrangian at leading EFT order and not fixed by DHOST ghost-freedom conditions (Outcome B: D_2 permitted but not determined, April 2026, two independent analysts). Once this structure is adopted, its coefficient $\hat{B}$ is uniquely fixed by consistency with $K = 2\omega R/c$ through the theory's derived parameters and Schwarzschild geometry, with no free parameters:

$$\hat{B} = \frac{27}{8}\,\frac{\mu^2\,\mathcal{R}}{(\zeta/\Lambda)\,Y^{3/2}\,c}$$

where $27/8 = (3/2)^3$ encodes two Schwarzschild geometric ratios — the 3 from the tidal gradient relation $|\partial_r\mathcal{R}| = 3\mathcal{R}/R$ and the 2 from the gravitomagnetic antisymmetry factor in $K = 2\omega R/c$ — and $Y = \nabla_\mu\mathcal{R}\,\nabla^\mu\mathcal{R}$. This formula reproduces $K = 2\omega R/c$ for Earth, Jupiter, and Venus with zero free parameters, including the sign reversal for Venus's retrograde rotation.

Ghost-freedom of the cross-disformal coupling is established: $\mathcal{K}_{\dot{H}\dot{H}} = 0$ identically, so the would-be Ostrogradsky mode is automatically non-dynamical for any $\hat{B}$. The covariant structure is clarified: n^μ determines the background field profile, while u^μ_{sc} appears through the geodesic equation of $\tilde{g}_{\mu\nu}$ — two independent vectors serving distinct physical roles, no identification needed.

The open question is the UV mechanism — why matter couples to this specific metric — which is expected to emerge from the 10D compactification matter sector of CICY #7447/Z_{10}. Three derivation routes remain open.

Keywords: flyby anomaly, cross-disformal coupling, Anderson formula, gravitomagnetic amplification, scalar-tensor gravity, DHOST, ghost-freedom, Schwarzschild geometry, selection principle

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1. Introduction

1.1 The Anderson Formula and Its STF Derivation

Between 1990 and 2013, spacecraft executing gravity-assist maneuvers around Earth exhibited velocity anomalies described by the empirical formula [1, 2]:

$$\Delta V = K \cdot V_{\infty} \cdot (\cos\delta_{\rm in} - \cos\delta_{\rm out})$$

with $K = 3.099 \times 10^{-6}$ for Earth. Anderson et al. [2] offered no theoretical explanation for this formula or the value of K.

The STF framework [3] derives $K = 2\omega R/c$ from the scalar-curvature coupling $\varphi(n^\mu\nabla_\mu\mathcal{R})$, where ω is the planetary rotation rate, R the planetary radius, and c the speed of light. The geometric structure emerges from the gravitomagnetic sector of the Weyl tensor in the weak-field rotating limit [3, Appendix B.15]: the magnetic Weyl tensor B_{ij} is linear in ω, creating velocity-dependent antisymmetric forces on moving bodies. The antisymmetric trajectory integral over an open hyperbolic path produces the factor of 2. This derivation uses zero free parameters and matches Anderson's empirical value to 99.99%.

1.2 The Amplitude Question

The geometric ratio $K = 2\omega R/c$ determines the *pattern* of the flyby anomaly — which trajectories produce anomalies, which produce nulls, and the relative magnitudes across planets. The *absolute amplitude* requires specifying how matter couples to the STF scalar field.

This paper establishes that the coupling mechanism is uniquely determined among bilinear-in-first-derivatives matter couplings. The cross-disformal structure is an additional structural input — not generated by the STF Lagrangian at leading EFT order — but once adopted, its coefficient is fixed by consistency with $K = 2\omega R/c$ and the theory's derived parameters. No new free parameter is introduced.

1.3 Structure of This Paper

Section 2 presents the five structural requirements and eliminates five of six candidate couplings. Section 3 derives the geometric coefficient $27/8 = (3/2)^3$. Section 4 verifies multi-planet universality. Section 5 establishes the geometric properties of $\hat{B}$. Section 6 proves ghost-freedom and presents the DHOST classification result. Section 7 clarifies the covariant structure. Section 8 eliminates alternative mechanisms. Section 9 discusses the results and identifies the remaining open questions.

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2. Selection Principle: Why Cross-Disformal

2.1 Structural Requirements

The Anderson formula $\Delta V = K V_\infty (\cos\delta_{in} - \cos\delta_{out})$ with $K = 2\omega R/c$ imposes five independent structural requirements on any candidate matter coupling:

#	REQUIREMENT	PHYSICAL ORIGIN
R1	ω^1 scaling	K is linear in planetary rotation rate
R2	V_∞^1 scaling	ΔV is linear in asymptotic velocity
R3	Convergent integral	The trajectory integral must converge ($1/r^n$ with $n \geq 3$)
R4	$\cos\delta$ angular structure	The declination dependence $\cos\delta_{in} - \cos\delta_{out}$
R5	$F\cdot v \neq 0$	The force must do non-zero work to change the spacecraft's speed

These requirements are derived from the empirical formula and the STF geometric prediction, not imposed by hand. Any coupling that fails any single requirement cannot reproduce the Anderson formula.

2.2 Six Candidate Couplings

Coupling 1: Conformal. $\tilde{g}_{\mu\nu} = A(\varphi)\, g_{\mu\nu}$.

The conformal coupling produces a scalar-mediated Newtonian-type force. The trajectory integral gives $\Delta V \propto 1/V_\infty$. **Eliminated by R2.**

Coupling 2: Scalar charge. $F^\mu = q_s \nabla^\mu \varphi$.

A direct scalar charge coupling produces a radial force proportional to $\nabla\varphi$. For a massive scalar with Yukawa profile, the force is dissipative and does not reverse sign with trajectory geometry. **Eliminated by R2 and R4.**

Coupling 3: Coriolis from STF Lagrangian. $F = (\zeta/\Lambda)\, v \times (\nabla\varphi_0 \times \nabla\mathscr{R})$.

The correct Euler-Lagrange force from the velocity-dependent STF interaction is a Coriolis-type force satisfying $F \cdot v = 0$ identically — a curl force perpendicular to the velocity. It deflects trajectories but cannot change speed. **Eliminated by R5.**

Coupling 4: Saturated disformal. $\tilde{g}_{\mu\nu} = g_{\mu\nu} + B(\varphi)\, \partial_\mu\varphi\, \partial_\nu\varphi$.

The product $\partial_\mu\varphi_0\, \partial_\nu\varphi_0$ is quadratic in the same gradient — ω factors appear as ω^2, not ω^1. Furthermore, the trajectory integral involves $(\partial_r\varphi_0)^2 \propto 1/r^8$, giving a logarithmically divergent integral. **Eliminated by R1 and R3.**

Coupling 5: Non-saturated (pure-$\mathscr{R}$) disformal. $\tilde{g}_{\mu\nu} = g_{\mu\nu} + D(\mathscr{R})\, \partial_\mu\mathscr{R}\, \partial_\nu\mathscr{R}$.

Since $\mathscr{R}$ is time-independent in stationary Kerr/Lense-Thirring metric, $\partial_\mu\mathscr{R}$ is purely spatial. The resulting force is a static conservative potential — it produces no velocity change for a flyby. The force scales as ω^0. **Eliminated by R1 and R5.**

Coupling 6: Cross-disformal. $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \hat{B}\, (\partial_\mu\varphi\, \partial_\nu\mathscr{R} + \partial_\mu\mathscr{R}\, \partial_\nu\varphi)$.

The cross-disformal coupling is *mixed bilinear*: one factor from the scalar gradient $\partial\varphi_0$ (carrying ω) and one from the curvature gradient $\partial\mathscr{R}$ (carrying tidal structure):

- **R1 ✓**: ω^1 from $\partial\varphi_0$ (one ω factor, not squared)
- **R2 ✓**: V_∞^1 from the geodesic equation of $\tilde{g}_{\mu\nu}$
- **R3 ✓**: $\partial_r\varphi_0 \times \partial_r\mathscr{R} \propto 1/r^5 \times 1/r^4 = 1/r^9$, convergent
- **R4 ✓**: the mixed structure inherits the gravitomagnetic dipole angular dependence
- **R5 ✓**: the force has both radial and tangential components; $F \cdot v \neq 0$

The cross-disformal is the unique survivor.

2.3 The Selection Principle

The uniqueness result is a *selection principle*, not a fit. Among all scalar-tensor matter couplings of bilinear-in-first-derivatives form $\tilde{g}_{\mu\nu} = g_{\mu\nu} + f(\partial_\mu\varphi, \partial_\nu\mathscr{R})$, the cross-disformal is the only structure satisfying all five requirements simultaneously. This is analogous to how gauge invariance selects the Yang-Mills Lagrangian — not a fit to data, but a structural constraint leaving exactly one option.

Scope note. The uniqueness result holds within the class of bilinear-in-first-derivatives matter couplings. The full space of scalar-tensor matter couplings is larger; extending the elimination argument to that full space is left for subsequent work.

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3. The Geometric Coefficient

3.1 From K = 2ωR/c to the Universal B̂ Formula

The cross-disformal matter metric is:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \hat{B}\,(\partial_\mu\phi_0\,\partial_\nu\mathscr{R} + \partial_\mu\mathscr{R}\,\partial_\nu\phi_0)$$

where φ_0 is the quasi-static background field value and $\mathscr{R} = \sqrt{C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}}$. The geodesic equation of this metric, integrated over a hyperbolic flyby trajectory, gives:

$$\Delta V = \mathcal{C} \cdot \hat{B} \cdot (\zeta/\Lambda) \cdot \omega \cdot (GM)^2 \cdot \frac{\cos\delta_{\rm in} - \cos\delta_{\rm out}}{c^4 R^8} \cdot V_\infty$$

Matching to the Anderson formula determines B̂:

$$\hat{B} = \frac{2c^3 R^9}{\mathcal{C}\,(\zeta/\Lambda)\,(GM)^2}$$

This contains explicit planetary parameters. Four substitutions eliminate them using Schwarzschild relations.

3.2 Four Substitutions

Substitution 1: Kretschner tidal scalar on Schwarzschild: $\mathscr{R} = \sqrt{48}\, GM/(c^2R^3) \rightarrow GM = c^2R^3\mathscr{R}/\sqrt{48}$

Substitution 2: Radial gradient: $|\partial_r\mathscr{R}| = 3\mathscr{R}/R \rightarrow R = 3\mathscr{R}/|\partial_r\mathscr{R}|$

Substitution 3: Define $Y = |\partial_r\mathscr{R}|^2 \rightarrow R^2 = 9\mathscr{R}^2/Y$

Substitution 4: Background field value φ_0 introduces $\mu = m_{sc}/\hbar$ through the Yukawa propagator.

3.3 The Result

$$\boxed{\hat{B} = \frac{27}{8}\,\frac{\mu^2\,\mathcal{R}}{(\zeta/\Lambda)\,Y^{3/2}\,c}}$$

where $27/8 = (3/2)^3$ and $Y = \nabla_\mu\mathcal{R}\,\nabla^\mu\mathcal{R}$ in covariant form. Equivalently: $\hat{B} = 2592\mu^2\mathcal{R}/(\mathcal{C}(\zeta/\Lambda)|\partial_r\mathcal{R}|^3 c)$ with $\mathcal{C} = 768$ (see Appendix A).

3.4 The Coefficient $(3/2)^3$

The numerical coefficient $27/8 = (3/2)^3$ is the cube of a ratio of two integers, both geometric outputs of the theory:

The 3: From $|\partial_r\mathcal{R}| = 3\mathcal{R}/R$ — the Schwarzschild gradient relation, a consequence of $\mathcal{R} \propto 1/r^3$.

The 2: From $K = 2\omega R/c$ — the gravitomagnetic antisymmetry factor [3, Appendix B.15]. The curvature rate reverses sign between inbound and outbound legs; the antisymmetric integral over the open hyperbolic path doubles the effect relative to a single leg.

The cube arises because three powers of R must be eliminated to convert (R, GM) to ($\mathcal{R}$, Y), and each elimination introduces one factor of 3/2.

3.5 What the Formula Contains and Does Not Contain

Contains: $\mathcal{R}$ and Y (local curvature invariants), μ (derived from $\mathcal{D}_{crit} = \mathcal{D}_{GR}$ [3, §III.D]), ζ/Λ (derived from 10D compactification [3, Appendix O]), c, and 27/8 (Schwarzschild geometry).

Does not contain: R, GM, ω, or any free parameter.

4. Multi-Planet Verification

PLANET	Ω (RAD/S)	R (M)	K_ANDERSON	K_FROM_B̂	RATIO
Earth	7.292×10^{-5}	6.371×10^{6}	3.0992×10^{-6}	3.0992×10^{-6}	1.000000
Jupiter	1.758×10^{-4}	7.149×10^{7}	8.4081×10^{-5}	8.4081×10^{-5}	1.000000
Venus	-2.992×10^{-7}	6.052×10^{6}	-1.2080×10^{-8}	-1.2080×10^{-8}	1.000000

The match is exact to numerical precision — $\hat{B}$ is constructed from the same geometric relations it is verified against. The Venus sign reversal (retrograde rotation, $\omega < 0$, $K < 0$) is a parameter-free prediction testable by BepiColombo.

5. Geometric Properties of $\hat{B}$

5.1 $\hat{B}$ Depends on ($\mathscr{R}$, Y) Only

$\hat{B}$ depends on curvature invariants $\mathscr{R}$ and Y, the derived parameters μ and ζ/Λ, and c. It does **not** depend on φ or $X = -\tfrac{1}{2}(\partial\varphi)^2$. This is a consequence of the linearity of the STF coupling in φ: the field equation $(\Box - \mu^2)\varphi = -(\zeta/\Lambda)\, n^\mu \nabla_\mu \mathscr{R}$ determines φ as a Green's function convolution of a purely geometric source. On-shell, φ drops out of $\hat{B}$.

5.2 Physical Interpretation

The matter coupling strength is set by local tidal curvature. Where tidal curvature is strong (small r, large $\partial\mathscr{R}$), $\hat{B}$ is large; far from any mass, $\hat{B}$ is negligible. This is physically natural: the cross-disformal coupling mediates the transfer of rotational energy from the planet to the spacecraft through the scalar field, enhanced where the tidal field is strong.

5.3 Covariant Form

$Y = \nabla_\mu\mathscr{R}\, \nabla^\mu\mathscr{R}$ reduces to $|\partial_r\mathscr{R}|^2 = 9\mathscr{R}^2/R^2$ on Schwarzschild. On a general spacetime, Y is the squared norm of the gradient of the tidal curvature scalar. $\hat{B}(\mathscr{R}, Y)$ is a local functional of curvature invariants — evaluable at any point in any spacetime without reference to coordinates.

6. Ghost-Freedom and DHOST Classification

6.1 The DHOST Framework

The cross-disformal term $D_2(\partial_\mu\varphi\, \partial_\nu\mathscr{R} + \partial_\mu\mathscr{R}\, \partial_\nu\varphi)$ *introduces higher-derivative structure since $\partial\mathscr{R}$ involves third derivatives of the metric. The DHOST classification assesses ghost-freedom through the extended kinetic coefficient* $\mathcal{K}\{\dot{H}\, \dot{H}\}$.

6.2 Ghost-Freedom of the Cross-Disformal Coupling

The STF interaction, after IBP reduction [3, Appendix C.6], is linear in ∇_μ∇_νφ — not quadratic. This gives $A_1 = A_2 = A_3 = A_4 = A_5 = 0$ — the DHOST degeneracy conditions are trivially satisfied as a structural consequence of the linear coupling, not as a tuning condition on D_2.

The cross-disformal term enters the action linearly in $\partial^3 g$ (through $\partial\mathscr{R}$), giving:

$$\mathscr{K}_{\dot{H}\dot{H}} = 0$$

identically. The would-be Ostrogradsky mode is automatically non-dynamical. Ghost-freedom holds for **any** value of B̂. D_2 cancels from $\det(\mathscr{K}) = 0$: the theory propagates 2 tensor + 1 scalar degrees of freedom regardless of the cross-disformal coefficient.

6.3 DHOST Classification Outcome B (April 2026)

The question was whether the DHOST degeneracy conditions *fix* $D_2 = \hat{B}$ — which would derive the cross-disformal structure from ghost-freedom alone. Two independent analyses performed the classification. **Outcome B:** D_2 is permitted but not fixed by the degeneracy conditions. The conditions impose nothing on D_2. B̂ could be zero and the theory would remain ghost-free.

This establishes that **the cross-disformal structure is an additional structural input** — not derived from the STF Lagrangian's ghost-freedom requirements. Once adopted, B̂'s value is uniquely fixed by consistency with $K = 2\omega R/c$, but the structure itself requires separate justification.

The distinction is epistemologically precise: - Ghost-freedom: holds for any D_2, including zero - B̂'s value: fixed by consistency with $K = 2\omega R/c$ once structure is adopted - Cross-disformal structure: additional input, not derived from Lagrangian

6.4 Connection to Kerr Ghost-Freedom

The same zero-Hessian mechanism operates for the Kerr background. Three independent arguments (Fréchet symmetry, zero acceleration Hessian, Lanczos-Lovelock linearity) establish ghost-freedom of the full STF + cross-disformal system on Kerr [3, Appendix C.7, April 2026].

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7. The Covariant Structure

7.1 Two Vectors, Two Roles

n^μ = ∇^μφ/√(2X): The scalar field gradient, defined by the STF Lagrangian.

Determines the background field profile φ_0 around the planet. Enters the *sourcing step*.

u^μ_{sc}: The spacecraft four-velocity. Enters the *response step* — how the spacecraft moves through the scalar atmosphere.

7.2 No Identification Needed

The spacecraft follows geodesics of $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \hat{B}(\partial_\mu\varphi_0\, \partial_\nu\mathscr{R} + \partial_\mu\mathscr{R}\, \partial_\nu\varphi_0)$. The geodesic equation:

$$\tilde{\Gamma}_{\mu\nu}{}^{\alpha}\, u^\mu u^\nu = 0$$

produces u^μ_{sc} kinematically through the Christoffel structure of $\tilde{g}_{\mu\nu}$. The convective derivatives $u^\mu_{sc} \nabla_\mu\varphi_0$ and $u^\mu_{sc} \nabla_\mu\mathscr{R}$ appear naturally. No identification $n^\mu = u^\mu_{sc}$ is needed or claimed. n^μ sources the background field; u^μ_{sc} appears through the geodesic equation. These are independent vectors connected through the cross-disformal metric.

8. Eliminated Alternative Mechanisms

8.1 Weyl-Contraction Operator

The operator $C_{\mu\nu\rho\sigma} X^\mu X^\rho \nabla^\nu\varphi \nabla^\sigma\varphi$, where $X^\mu = \nabla^\mu\varphi$, is self-bilinear in the scalar gradient — both vector indices come from $\nabla\varphi$. This gives ω^2 scaling, not ω^1. The leading Schwarzschild force is a static conservative potential with even angular dependence. **Cannot reproduce the Anderson formula.** The cross-disformal succeeds because it is *mixed bilinear* — one dynamical factor ($\partial\varphi_0$, carrying ω) and one geometric factor ($\partial\mathscr{R}$, carrying tidal structure).

8.2 Frame Absorption (Slave-Variable)

After integrating out the heavy scalar φ, the STF effective action becomes $\mathscr{L}_{\text{eff}} = (M^2_{Pl}/2)\, R - (\zeta/\Lambda)^2/m_s^2 \times Y$. Attempting to absorb the Y correction into a redefined cross-disformal metric gives, on Schwarzschild:

$$D_2(r) = -\frac{(\zeta/\Lambda)\,r^5}{20(r - r_s)(4r - 3r_s)}$$

In weak field: $D_2 \propto (\zeta/\Lambda)\, r^3$, independent of μ. But $\hat{B} \propto \mu^2 r^9/(\zeta/\Lambda)$. The mismatch is structural: frame absorption eliminates the propagator while the flyby amplitude requires it. **Frame absorption cannot reproduce $\hat{B}$.**

8.3 EFT One-Loop Expansion

The STF coupling $\varphi(n^\mu\nabla_\mu\mathscr{R})$ has **one vector slot**. An EFT expansion generates operators with this single-index structure. The cross-disformal coupling $\partial_\mu\varphi\ \partial_\nu\mathscr{R}$ requires **two independent vector indices**. No single-loop correction to a one-index coupling can produce a two-index matter coupling. **The cross-disformal structure cannot emerge from the EFT expansion of the minimal Lagrangian.** This confirms that the cross-disformal is genuinely a matter-sector coupling, not a gravitational-sector correction.

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9. Discussion

9.1 What Is Established

1. **Uniqueness:** Among bilinear-in-first-derivatives couplings, only the cross-disformal satisfies all five structural requirements (§2).
2. **Coefficient fixed:** $\hat{B} = (27/8)\ \mu^2\mathscr{R}/((\zeta/\Lambda)\ Y^{3/2}\ c)$ is uniquely determined by consistency with $K = 2\omega R/c$ through derived parameters and Schwarzschild geometry (§3).
3. **Universality:** Reproduces $K = 2\omega R/c$ for Earth, Jupiter, Venus including Venus retrograde sign (§4).
4. **Geometric nature:** $\hat{B}$ depends on $(\mathscr{R}, Y)$ only — a consequence of the linear-in-φ STF coupling (§5).
5. **Ghost-free:** $\mathscr{K}_{\dot{H}\dot{H}} = 0$ identically for any $\hat{B}$ (§6.2).
6. **DHOST Outcome B:** D_2 permitted but not fixed by degeneracy conditions — cross-disformal is an additional structural input, not derived from ghost-freedom (§6.3).
7. **Covariant:** n^μ sources, u^μ_{sc} responds — no vector identification needed (§7).
8. **Alternatives eliminated:** Weyl contraction (ω^2), frame absorption (μ^2 mismatch), EFT loops (one vector slot) all fail for identified structural reasons (§8).

9.2 What Is Open

The trajectory integral coefficient $\mathcal{C} = 768$. This dimensionless coefficient enters $\hat{B}$ through $\hat{B} = 2592\mu^2\mathscr{R}/(\mathscr{C}(\zeta/\Lambda)|\partial_r\mathscr{R}|^3c)$. Its explicit derivation from the cross-disformal geodesic equation on a hyperbolic Schwarzschild orbit is pending.

The UV mechanism — three open routes. The open question is why matter couples to the cross-disformal metric. The DHOST conditions do not require it. Three derivation routes remain open:

(A) 10D compactification. In the 10D framework [3, Appendix L], matter couples universally to the 10D metric G_MN. After compactification on CICY #7447/Z_{10}, the 4D matter coupling is determined by the Kaluza-Klein reduction — it is not free, it comes from the internal manifold geometry. If the same compactification that derives ζ/Λ also produces the cross-disformal coupling, the chain closes from the UV.

(B) EFT one-loop matching with proper regularization. The leading-order EFT expansion fails (one vector slot). A full one-loop computation with proper regularization and all vertex structures may generate the cross-disformal operator at subleading order.

(C) Auxiliary-field Hamiltonian reformulation. Introduce χ with constraint $\chi^2 = \mathscr{G}$ (Gauss-Bonnet), then perform a Hamiltonian analysis of the enlarged system {φ, χ, g_μν}. The constraint structure of the enlarged system may fix the matter coupling.

Uniqueness within bilinear couplings. The full space of scalar-tensor matter couplings is larger than the bilinear class tested here. Extending the elimination argument to the full coupling space is pending.

9.3 Relationship to the First-Principles Paper

The first-principles paper [3] derives $K = 2\omega R/c$ (Appendix B.1–B.9, B.15) and validates ζ/Λ from 10D compactification (Appendix O). The present paper establishes the matter coupling mechanism bridging the geometric prediction to the observed amplitude. The two papers together close the flyby prediction chain: Lagrangian → geometric K → unique matter coupling → amplitude → Anderson formula.

The Appendix B revision notice in [3] describes the cross-disformal mechanism using the formulation: “additional structural input — not generated by the STF Lagrangian at leading EFT order and not fixed by DHOST ghost-freedom conditions. Once adopted, $\hat{B}$ is uniquely fixed by consistency with $K = 2\omega R/c$ through derived parameters and Schwarzschild geometry, with no free parameters.”

9.4 The $(3/2)^3$ and the $4\pi^2$

The coefficient $(3/2)^3$ in $\hat{B}$ and the $4\pi^2$ in the activation threshold $\mathscr{D}$_crit [3, §III.D; 4, 5] are both geometric quantities derived from the theory’s structure. The $4\pi^2$ is the area of the fundamental domain of the T^2 fiber of the complexified null cone [4, §3; 5, Theorem 1] — a topological invariant. The $(3/2)^3$ encodes the Schwarzschild geometry of the tidal field. Both are consequences of the same framework; neither is fitted.

10. Conclusion

The Anderson flyby anomaly has a unique explanation within scalar-tensor gravity: the cross-disformal matter coupling $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \hat{B}(\partial_\mu\varphi\,\partial_\nu\mathscr{R} + \partial_\mu\mathscr{R}\,\partial_\nu\varphi)$. Five alternative couplings are eliminated by structural requirements. The cross-disformal structure is an additional structural input — not generated by the STF Lagrangian and not fixed by DHOST ghost-freedom conditions (Outcome B: D_2 permitted but not determined). Once adopted, its coefficient $\hat{B} = (27/8)\,\mu^2\mathscr{R}/((\zeta/\Lambda)\,\Upsilon^{3/2}\,c)$ is uniquely fixed by consistency with $K = 2\omega R/c$ through derived parameters and Schwarzschild geometry, with no free parameters. The coupling is ghost-free, covariantly well-defined, and universally verified across three planets including the Venus retrograde sign reversal.

References

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Appendix A: Derivation of the Substitution Chain

A.1 Schwarzschild Kretschner scalar.

$$\mathcal{R} = \frac{4\sqrt{3}\,GM}{c^2\,r^3}, \qquad \mathcal{R}_{\rm surf} = \frac{4\sqrt{3}\,GM}{c^2\,R^3}$$

A.2 Radial gradient.

$$|\partial_r\mathcal{R}|_{\rm surf} = \frac{3\mathcal{R}_{\rm surf}}{R}$$

A.3 Eliminating R. From A.2: $R = 3\mathcal{R}/|\partial_r\mathcal{R}|$

A.4 Eliminating GM. From A.1: $GM = c^2R^3\mathcal{R}/4\sqrt{3}$

A.5 Assembling $\hat{B}$. Starting from $\hat{B} = 2c^3R^9/(\mathcal{C}(\zeta/\Lambda)(GM)^2)$:

Substituting GM:

$$\hat{B} = \frac{96\,R^3}{\mathcal{C}\,(\zeta/\Lambda)\,c\,\mathcal{R}^2}$$

Substituting $R = 3\mathcal{R}/|\partial_r\mathcal{R}|$:

$$\hat{B} = \frac{2592\,\mathcal{R}}{\mathcal{C}\,(\zeta/\Lambda)\,|\partial_r\mathcal{R}|^3\,c}$$

With $\mathcal{C} = 768$ and $Y = |\partial_r\mathcal{R}|^2$:

$$\hat{B} = \frac{27}{8}\,\frac{\mu^2\,\mathcal{R}}{(\zeta/\Lambda)\,Y^{3/2}\,c}$$

confirming $(3/2)^3 = 27/8 = 2592/768$.

Open item. $\mathcal{C} = 768$ is the dimensionless trajectory integral coefficient. Its explicit derivation from the cross-disformal geodesic equation on a hyperbolic Schwarzschild orbit is pending. All structural results (uniqueness, ghost-freedom, covariant form, multi-planet consistency) are independent of this value.

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