Cross-Messenger Clock Closure in Two-Clock Cosmology

A universal-lapse null theorem and an STF propagation template

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Companion code: `stf_cross_messenger_clock_closure_checks.py`

Abstract

We introduce a cross-messenger clock-closure observable that compares the redshift drift inferred from a long gravitational-wave chirp with the redshift drift measured

electromagnetically for the same cosmological source population. For an FLRW spacetime written with an arbitrary universal lapse, and for electromagnetic and gravitational signals sharing the same null cone, the two measurements obey an exact kinematic identity. Defining

$$X_{\mathrm{EM}}(z) \equiv \frac{\dot{z}_{\mathrm{EM}}}{2(1+z)}, \qquad \mathcal{R}_{2\mathrm{C}}(z) \equiv X_{\mathrm{GW}}(z) - X_{\mathrm{EM}}(z),$$

we prove that

$$\boxed{\mathcal{R}_{2\mathrm{C}}(z) = 0}$$

for every common-lapse realization, independently of the functional form of the lapse. A nonzero residual therefore cannot be attributed to a mere universal reparametrization of cosmological time. It requires differential propagation, nonuniversal clock coupling, or an unmodelled systematic.

For a weak time-dependent tensor speed, $c_T = 1 + \delta c_T$, the leading local residual is

$$\boxed{\mathcal{R}_{2\mathrm{C}} = \frac{1}{2}\left[\frac{\dot{\delta c}_{T,e}}{1+z} - \dot{\delta c}_{T,o}\right]}$$

to first order in δc_T. We also derive the corresponding finite-baseline endpoint phase. Applied conditionally to the selective transient field (STF) scalar–Gauss–Bonnet propagation branch, the result is an oscillatory source–observer template governed by the internal scalar phase, while the cosmological acceleration term is governed by the universal clock. The endpoint sinusoid itself is shared with recent ultralight-scalar propagation work; the distinctive proposal here is the cross-messenger closure relation and its universal-lapse invisibility theorem.

Using the response-matched coefficient and scalar benchmark recorded in STF First-Principles V8.1, the instantaneous tensor-speed envelope is 3.6×10^{-30}, the internal period is 3.326 yr, and the maximum closure residual at $z = 1$ is approximately $2.60 \times 10^{-37}\,\mathrm{s}^{-1}$, about 2.27×10^{-18} of the standard ΛCDM acceleration signal. This benchmark is not observationally competitive. The main result is instead a clean null test that separates universal-clock effects from genuinely messenger-dependent propagation and supplies a falsifiable interface for future two-clock completions.

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1. Scope and status

The selective transient field framework distinguishes two time structures:

1. a **universal clock**, represented covariantly by a timelike foliation or clock field and governing causal relaxation, cosmological evolution, and the common time standard used by matter; and
2. an **internal clock**, supplied by the phase of an ultralight oscillating scalar and governing periodic response.

The question addressed here is deliberately narrower than the construction of a complete STF action:

> Can the two-clock distinction be turned into an operational, cross-messenger null test that remains meaningful even before the full nonlinear closure sector is completed?

The answer is yes. The test rests on a kinematic theorem and therefore does not require the unresolved nonlinear parent completion. Its mapping to the specific STF scalar–Gauss–Bonnet branch is conditional on that branch being part of the final healthy parent theory.

This paper makes four claims, with different logical status:

- **Theorem:** an arbitrary lapse common to photons, gravitational waves, matter clocks, and the background expansion cancels from the cross-messenger closure residual.
- **First-order propagation result:** a weak messenger-dependent tensor speed produces a residual equal to the emitter contribution divided by $1 + z$, minus the observer contribution.
- **Conditional STF template:** the response-matched STF scalar–Gauss–Bonnet branch generates a periodic endpoint phase and closure residual with the internal scalar period.
- **Benchmark conclusion:** the coefficient set recorded in STF V8.1 makes this cosmological effect extraordinarily small; the observable is currently more valuable as a structural null test than as a near-term detection forecast.

We do **not** claim in this paper that the full STF parent action has been completed, that the cosmological split-leg closure has passed every scalar constraint test, or that a detectable tensor-speed signal follows from the dark-matter capacity coefficient. In particular, the bare support coefficient $\kappa \sim 10^{70}$ is not substituted into the metric propagation leg. That substitution was an invalid identification in an earlier exploratory calculation and is excluded here.

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2. Operational clock map

Three time variables appear in the measurement, although only two are fundamental STF clock structures.

SYMBOL	OPERATIONAL ROLE	STF INTERPRETATION
τ_o	detector proper time and electromagnetic time standard	realization of the universal clock at the observer

SYMBOL	OPERATIONAL ROLE	STF INTERPRETATION
τ_e	source proper time	realization of the same universal clock at emission
Θ_I	phase of the oscillating scalar	internal clock

The distinction is not between two coordinate labels. It is between a shared causal/proper-time structure and a dynamical phase that can vary relative to it. A lapse redefinition changes how a coordinate T parametrizes the universal clock, but cannot by itself create a difference between two messengers that couple to that same clock.

We write a spatially flat homogeneous metric as

$$ds^2 = -L_U^2(T)dT^2 + a^2(T)\left(d\chi^2 + \chi^2 d\Omega^2\right), \qquad d\tau = L_U(T)dT.$$

The physical Hubble rate is

$$H \equiv \frac{1}{a}\frac{da}{d\tau} = \frac{1}{L_U}\frac{1}{a}\frac{da}{dT}.$$

The function $L_U(T)$ may be nontrivial. No choice of time coordinate or gauge is made in the theorem below.

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3. Electromagnetic redshift drift with an arbitrary lapse

For a radial electromagnetic null ray,

$$0 = -L_U^2 dT^2 + a^2 d\chi^2,$$

so a comoving source at fixed χ satisfies

$$\chi = \int_{T_e}^{T_o} \frac{L_U(T)}{a(T)} dT.$$

Varying the endpoints for two neighboring wavefronts gives

$$\frac{L_o}{a_o} dT_o = \frac{L_e}{a_e} dT_e.$$

Since $d\tau = L_U dT$, this becomes

$$\frac{d\tau_o}{a_o} = \frac{d\tau_e}{a_e}, \qquad 1 + z = \frac{d\tau_o}{d\tau_e} = \frac{a_o}{a_e}.$$

Differentiate with respect to observer proper time:

$$\frac{d}{d\tau_o} \ln(1+z) = H_o - \frac{H_e}{1+z}.$$

Therefore

$$\dot{z}_{\rm EM} = (1+z)H_o - H_e,$$

and

$$\boxed{X_{\rm EM}(z) \equiv \frac{\dot{z}_{\rm EM}}{2(1+z)} = \frac{1}{2}\left(H_o - \frac{H_e}{1+z}\right).}$$

Every explicit lapse factor has disappeared. The physical expansion rate retains whatever dynamics the theory assigns to it, but an arbitrary coordinate lapse does not become a separately measurable redshift-drift signal.

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4. The universal-lapse closure theorem

Theorem 1 — Common-lapse messenger closure

Assume:

1. photons and high-frequency tensor waves propagate on the same null cone;
2. emission and reception frequencies are measured using matter clocks coupled to the same physical metric;
3. geometric optics is valid between the source and detector; and
4. source peculiar acceleration, local gravitational potentials, and line-of-sight structure are either negligible or included identically in the two messenger models.

Then, for an arbitrary lapse $L_U(T)$,

$$X_{\mathrm{GW}}(z) = X_{\mathrm{EM}}(z),$$

and hence

$$\boxed{\mathcal{R}_{2\mathrm{C}}(z) = 0.}$$

Proof

Under the assumptions, the gravitational-wave eikonal obeys the same null equation as the electromagnetic eikonal. The endpoint variation is therefore

$$\frac{L_o}{a_o} dT_o = \frac{L_e}{a_e} dT_e$$

for both messengers. Converting both endpoints to the same physical proper times gives

$$1 + z_{\mathrm{GW}} = 1 + z_{\mathrm{EM}} = \frac{a_o}{a_e}.$$

Differentiation with respect to detector proper time yields the same drift,

$$\frac{\dot{z}_{\mathrm{GW}}}{1+z} = \frac{\dot{z}_{\mathrm{EM}}}{1+z} = H_o - \frac{H_e}{1+z}.$$

The difference vanishes. $\square$

Corollary 1 — Universal clock invisibility

A universal modification of the time parametrization, including a nontrivial common lapse, may change the background solution $a(\tau)$, but it cannot generate a messenger difference at fixed physical expansion history. The closure residual is blind to universal time reparametrization.

Corollary 2 — Meaning of a nonzero residual

After controlled systematics are removed, $\mathcal{R}_{2\mathrm{C}} \neq 0$ implies that at least one assumption of the theorem fails. Possibilities include a distinct tensor cone, nonuniversal matter coupling, frequency-dependent propagation, phase-dependent endpoint physics, or a breakdown of the assumed homogeneous propagation model.

This is the main reason the test is useful. It is a discriminator, not merely another measurement of $H(z)$.

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5. How the gravitational-wave chirp measures X_{GW}

For a quasi-circular inspiral, let $\mathcal{M}_z = (1+z)\mathcal{M}$ be the redshifted chirp mass and define the leading time remaining to coalescence at observed frequency f:

$$\Delta T(f) = \frac{5}{256}\mathcal{M}_z^{-5/3}(\pi f)^{-8/3},$$

in units $G = c = 1$. A slowly varying redshift produces the Fourier-phase correction

$$\boxed{\Psi_{\mathrm{acc}}(f) = +2\pi f X_{\mathrm{GW}} \Delta T^2(f) = +\frac{25}{32768} X_{\mathrm{GW}} \mathcal{M}_z^{-10/3}(\pi f)^{-13/3}.}$$

The positive sign is important. Liu, Feng, and Guo rederived this term by matched asymptotic expansion in 2025 and corrected the opposite sign used in part of the earlier literature. We adopt their convention throughout. Changing the Fourier-transform convention flips all Fourier phases together but does not alter the closure identity.

The EM-side quantity is measured independently:

$$X_{\mathrm{EM}}(z) = \frac{1}{2(1+z)}\frac{dz}{d\tau_o}.$$

In practice, the two measurements need not come from the same individual object. One may compare a hierarchical GW inference of $X_{\mathrm{GW}}(z)$ with a redshift-drift reconstruction in matched redshift bins. A genuinely simultaneous standard-siren counterpart is nevertheless valuable because it constrains the redshift, peculiar acceleration, host environment, and line-of-sight nuisance model.

The observable is then

$$\boxed{\mathcal{R}_{2\mathrm{C}}(z) = X_{\mathrm{GW}}(z) - \frac{\dot{z}_{\mathrm{EM}}(z)}{2(1+z)}.}$$

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6. Differential tensor propagation

Now let gravitational waves propagate with a weak time-dependent speed

$$c_T(t) = 1 + \delta c_T(t), \qquad |\delta c_T| \ll 1,$$

while photons remain luminal in the matter frame. For neighboring tensor wavefronts,

$$\frac{c_{T,o}}{a_o}dt_o = \frac{c_{T,e}}{a_e}dt_e.$$

The gravitational-wave redshift is therefore

$$1 + z_{\mathrm{GW}} = \frac{dt_o}{dt_e} = \frac{a_o}{a_e}\frac{c_{T,e}}{c_{T,o}}.$$

Taking an observer-time derivative gives the exact geometric-optics relation

$$\frac{d}{dt_o}\ln(1+z_{\rm GW}) = H_o - \frac{H_e}{1+z_{\rm GW}} + \frac{1}{1+z_{\rm GW}}\frac{\dot c_{T,e}}{c_{T,e}} - \frac{\dot c_{T,o}}{c_{T,o}}.$$

Expanding to first order in δc_T and comparing with the EM drift yields

$$\boxed{\mathcal{R}_{\rm 2C}(z) = \frac{1}{2}\left[\frac{\dot{\delta c}_{T,e}}{1+z} - \dot{\delta c}_{T,o}\right] + \mathcal{O}(\delta c_T^2).}$$

The ordering is **emitter minus observer**. This corrects the sign ordering in the exploratory conversation that motivated the present paper. The magnitude estimates made there are unchanged, but the phase labels are not.

6.1 Finite-baseline endpoint phase

The local drift formula is the short-baseline expansion of a more general endpoint effect. For fixed comoving distance,

$$\chi = \int_{t_e}^{t_o} \frac{1+\delta c_T(t)}{a(t)}dt.$$

Relative to luminal propagation, the arrival-time perturbation is

$$\delta t_{\rm prop} = -\int_{t_e}^{t_o} \frac{\delta c_T(t)}{a(t)}dt + \mathcal{O}(\delta c_T^2).$$

For an oscillatory speed

$$\delta c_T(t) = \epsilon(t)\cos\Theta_I(t), \qquad \dot\Theta_I = m_s,$$

and amplitudes varying slowly compared with m_s^{-1}, integration by parts gives

$$\delta t_{\rm prop} \simeq \frac{1}{m_s}[-\epsilon_o\sin\Theta_o + (1+z)\epsilon_e\sin\Theta_e].$$

With the Fourier convention used above,

$$\boxed{\delta\Psi_T(f) = \frac{2\pi f}{m_s}[-\epsilon_o \sin\Theta_o(f) + (1+z)\epsilon_e \sin\Theta_e(f)].}$$

The frequency-dependent endpoint phases are

$$\Theta_o(f) = \Theta_{c,o} - m_s \Delta T(f),$$

$$\Theta_e(f) = \Theta_{c,e} - \frac{m_s \Delta T(f)}{1+z}.$$

This exact leading endpoint template is preferable whenever the observation spans an appreciable fraction of the internal period. Expanding both sines through second order in $m_s \Delta T$ produces:

- an f-proportional term degenerate with coalescence time;
- an $f\Delta T \propto f^{-5/3}$ term that can mix with intrinsic chirp parameters; and
- an $f\Delta T^2 \propto f^{-13/3}$ term whose coefficient is precisely $\mathcal{R}_{2\mathrm{C}}$.

Explicitly,

$$\delta\Psi_T^{(2)} = 2\pi f \Delta T^2 \frac{1}{2}\left[\frac{\delta\dot{c}_{T,e}}{1+z} - \delta\dot{c}_{T,o}\right].$$

Thus the endpoint calculation and the redshift-drift calculation agree.

6.2 What is and is not new

Oscillating ultralight fields coupled through parity-even Gauss–Bonnet terms are already known to generate endpoint-dependent gravitational-wave speed, redshift, amplitude, and phase effects. Jenks and Kamionkowski developed this physics explicitly for the axiverse in 2026. The sinusoidal endpoint structure should therefore not be presented as uniquely STF.

The new object proposed here is the closure construction:

$$\text{GW chirp drift} - \text{EM redshift drift},$$

together with the proof that an arbitrary universal lapse cancels exactly. The STF contribution is a specific interpretation and coefficient map for one conditional propagation branch.

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7. Conditional scalar–Gauss–Bonnet map

Consider the parity-even tensor propagation sector

$$S \supset \int d^4x\sqrt{-g}\,F(\phi)\,\mathcal{G},$$

where $\mathcal{G}$ is the Gauss–Bonnet invariant. To keep sign and action-normalization conventions explicit, write the quadratic tensor coefficients as

$$Q_T = M_{\rm Pl}^2 + 8\sigma_{\rm GB}H\dot{F}, \qquad F_T = M_{\rm Pl}^2 + 8\sigma_{\rm GB}\ddot{F},$$

so

$$c_T^2 = \frac{F_T}{Q_T}, \qquad \delta c_T \simeq \frac{4\sigma_{\rm GB}}{M_{\rm Pl}^2}(\ddot{F} - H\dot{F}).$$

Here $\sigma_{\rm GB} = \pm 1$ records the curvature and coupling-sign convention. It can equivalently be absorbed into F. The no-ghost and no-gradient conditions are

$$Q_T > 0, \qquad F_T > 0.$$

For a linear coupling and a coherently oscillating scalar,

$$F(\phi) = F_{,\phi}\phi, \qquad \phi(t) = \Phi(t)\cos\Theta_I(t), \qquad \dot{\Theta}_I = m_s,$$

with $m_s \gg H$,

$$\delta c_T \simeq -\frac{4\sigma_{\rm GB}F_{,\phi}m_s^2\Phi}{M_{\rm Pl}^2}\cos\Theta_I.$$

Therefore

$$\delta\dot{c}_T \simeq \frac{4\sigma_{\rm GB}F_{,\phi}m_s^3\Phi}{M_{\rm Pl}^2}\sin\Theta_I,$$

up to relative corrections of order H/m_s. The two-clock residual becomes

$$\boxed{\mathcal{R}_{\rm 2C}(z) \simeq \frac{2\sigma_{\rm GB}F_{,\phi}m_s^3}{M_{\rm Pl}^2}\left[\frac{\Phi_e\sin\Theta_e}{1+z} - \Phi_o\sin\Theta_o\right].}$$

For a coherently oscillating field that supplies a matter-like mean density,

$$\Phi_e = \Phi_o(1+z)^{3/2},$$

so

$$\boxed{\mathcal{R}_{\rm 2C}(z) \simeq \frac{2\sigma_{\rm GB}F_{,\phi}m_s^3\Phi_o}{M_{\rm Pl}^2}\left[\sqrt{1+z}\sin\Theta_e - \sin\Theta_o\right].}$$

The endpoint phase is

$$\delta\Psi_T(f) \simeq \frac{8\pi\sigma_{\rm GB}F_{,\phi}m_s f}{M_{\rm Pl}^2}\left[\Phi_o\sin\Theta_o(f) - (1+z)\Phi_e\sin\Theta_e(f)\right].$$

These equations exhibit the two clocks directly:

- $\Delta T(f)$, z, and $X_{\rm EM}$ are tied to the universal cosmological clock;
- $\Theta_I = m_s\tau + \Theta_0$ is the internal scalar clock.

A common lapse alters neither their closure theorem nor the messenger difference. A differential propagation coefficient allows the internal phase to appear in the GW channel without appearing in the EM redshift drift.

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8. STF coefficient ledger

The STF source record contains several coefficients with different physical roles. Keeping them separate is essential.

8.1 Support/capacity coefficient

The parent-level closure analysis defines

$$Q = P_A \mathcal{C}^A = M_*^2 q_N,$$

with first-order memory

$$(D_U + \omega_c) y = \omega_c Q, \qquad Z = Q - y,$$

and high-pass response

$$K^R_{\rm sel}(\omega) = \frac{-i\omega}{\omega_c - i\omega}.$$

The scalar support normalization is controlled by

$$\kappa = \gamma \tau_{\rm eff} M_*^2 = \frac{\zeta/\Lambda}{L_*^2}.$$

This coefficient belongs to the saturated closure readout. It is not automatically the coefficient of the metric tensor-propagation leg.

8.2 Response-matched metric coefficient

The V8.1 tensor-speed estimate uses a separately response-matched coupling

$$F_{,\phi} = c_{\rm GB} \left(\frac{\zeta}{\Lambda}\right) M_{\rm Pl}, \qquad c_{\rm GB} = \mathcal{O}(1).$$

The symbol $c_{\rm GB}$ is used here to avoid collision with the large support coefficient κ. This distinction removes the erroneous 10^{-80} tensor-speed estimate and the associated 10^{68}

coefficient-gap claim from the earlier exploratory branch.

8.3 Memory coefficient

If the metric propagation leg is itself filtered by the closure memory, its Fourier-domain amplitude acquires an additional factor

$$K^{R}_{\rm sel}(\omega) = \frac{-i\omega}{\omega_c - i\omega}.$$

At the STF benchmark $\omega \sim \omega_c$, the derivative expansion is not controlled and the exact kernel must be retained. Whether this factor multiplies the cosmological scalar–Gauss–Bonnet leg is a parent-theory question; it is not assumed in the numerical benchmark below.

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9. Benchmark

We use the parameters recorded in STF First-Principles V8.1:

$$m_s = 3.94 \times 10^{-23}\ \mathrm{eV},$$

$$\omega_s = \frac{m_s}{\hbar} = 5.9859 \times 10^{-8}\ \mathrm{s}^{-1},$$

$$T_s = \frac{2\pi}{\omega_s} = 3.32618\ \mathrm{yr},$$

and the instantaneous tensor-speed envelope

$$|\delta c_T|_{\rm env} = 3.6 \times 10^{-30}.$$

For a flat ΛCDM reference cosmology with

$$H_0 = 67.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}, \quad \Omega_m = 0.315, \quad \Omega_\Lambda = 0.685,$$

the electromagnetic acceleration variable at $z = 1$ is

$$X_{\rm EM}(1) = 1.14538 \times 10^{-19}\ \mathrm{s}^{-1}.$$

If $\Phi_e = \Phi_o(1+z)^{3/2}$, the phase-independent maximum of the closure residual is

$$|\mathcal{R}_{2\rm C}|_{\max} = \frac{\omega_s}{2} |\delta c_T|_{o,\rm env} \left(1 + \sqrt{1+z}\right).$$

At $z = 1$,

$$\boxed{|\mathcal{R}_{2\rm C}|_{\max} = 2.60123 \times 10^{-37}\ \mathrm{s}^{-1}}$$

and

$$\boxed{\frac{|\mathcal{R}_{2\rm C}|_{\max}}{X_{\rm EM}} = 2.27107 \times 10^{-18}.}$$

This is not a plausible detection target with the present response-matched coefficient.

9.1 Inspiral phase scale

For a source-frame binary-neutron-star chirp mass $\mathcal{M} = 1.22 M_\odot$ at $z = 1$, $\mathcal{M}_z = 2.44 M_\odot$. The reproducibility script gives:

F (HZ)	ΔT (YR)	$\Psi_{\rm ACC}$ (RAD)	$\lvert\Delta\Psi_T\rvert$ ENDPOINT BOUND (RAD)
0.05	13.6639	6.6904×10^{-3}	1.2577×10^{-22}
0.10	2.15193	3.3189×10^{-4}	2.5155×10^{-22}
0.20	0.338907	1.6464×10^{-5}	5.0310×10^{-22}

The acceleration phase is small but conceptually measurable through a population of long-lived decihertz sources. The STF propagation phase at this benchmark is about nineteen orders of magnitude smaller.

9.2 Interpretation

The tiny benchmark has two implications:

1. a null result at foreseeable sensitivity does not constrain the existing V8.1 response-matched branch; and
2. any future detectable closure residual would require a different coefficient regime, a resonance, an activation mechanism, an environmental enhancement, or physics beyond the present branch.

The observable remains useful because its common-lapse null prediction is exact. A measurement can be reported independently of whether the STF benchmark is large enough to explain it.

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10. Data-analysis form of the test

10.1 Minimal phenomenological template

A model-independent analysis can fit

$$\Psi(f) = \Psi_{\mathrm{src}}(f) + 2\pi f\,[X_{\mathrm{EM}}(z) + \mathcal{R}_{2\mathrm{C}}(z)]\Delta T^2(f),$$

with X_{EM} constrained externally and $\mathcal{R}_{2\mathrm{C}}$ treated as a free parameter in each redshift bin.

This is appropriate when the differential propagation changes slowly over the observed chirp. It directly tests the universal-lapse null theorem.

10.2 Oscillatory endpoint template

For an internal clock with a period comparable to the mission duration, the local expansion is insufficient. The fit should instead include

$$\delta\Psi_T(f) = \frac{2\pi f}{m_s}\left[-\epsilon_o \sin(\Theta_{c,o} - m_s\Delta T) + (1+z)\epsilon_e \sin\left(\Theta_{c,e} - \frac{m_s\Delta T}{1+z}\right)\right].$$

The source and observer phases should initially be nuisance parameters. A coherent Galactic or cosmological scalar model may correlate them, but that correlation must be justified using the scalar coherence length and propagation history rather than assumed.

10.3 Population hierarchy

A practical hierarchical model separates:

$$X_{\mathrm{GW}}(z, \hat{n}, t_o) = X_{\mathrm{cos}}(z) + X_{\mathrm{pec}} + X_{\mathrm{env}} + X_{\mathrm{lens}} + X_{\mathrm{prop}},$$

where:

- X_{cos} is the homogeneous expansion drift;
- X_{pec} is source/observer peculiar acceleration;
- X_{env} contains binary center-of-mass acceleration and host potential effects;
- X_{lens} represents line-of-sight time-dependent structure; and
- X_{prop} is the candidate differential propagation term.

The EM analysis must use the corresponding source and line-of-sight model. A clean cosmological residual is expected to be coherent in redshift and, for the oscillatory STF template, periodic in observer time with the same m_s across sources.

10.4 Discriminants

The following patterns help separate a two-clock propagation signal from nuisance effects:

- **Universal period:** a common multi-year frequency across independent sources.
- **Endpoint structure:** a detector-phase term shared by all sources plus an emitter term with a redshifted phase argument.
- **Redshift scaling:** the matter-like STF branch predicts $\Phi_e/(1+z) \propto \sqrt{1+z}$.

- **Messenger selectivity:** the periodic term appears in the GW closure residual but not in the EM drift.
- **Sky coherence:** a local observer field produces a monopolar time dependence, while source fields can decorrelate over large separations.
- **Polarization:** the parity-even Gauss–Bonnet branch is nonbirefringent at leading order, unlike a Chern–Simons branch.

No single event is expected to establish the effect. The test is naturally hierarchical and cross-messenger.

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11. Relation to the STF parent-completion audits

The clock-closure theorem is kinematic. The STF interpretation is not. It inherits the current conditional status of the split-leg completion.

11.1 What has passed

The archived parent-level audits establish the following limited results:

- the causal memory equation

  $$(D_U + \omega_c)y = \omega_c Q$$

  has a relaxation pole rather than an Ostrogradsky pole;
- the memory first-order variables form a second-class pair that removes the redundant canonical momenta;
- on conformally flat FLRW, the normalized curvature readout has no linear transverse-traceless perturbation, $\delta q_N^{TT} = 0$;
- the closure dust sector has no linear TT anisotropic stress;

- the first-order parent memory therefore does not add a tensor Ostrogradsky pair in the tested FLRW-TT sector; and
- the Einstein–canonical scalar baseline remains regular at internal-clock turning points in flat gauge.

11.2 What remains conditional

The same audit chain also found real obstructions:

- a local reduced reciprocal functional $F(q_N[g])$ generically produces a higher-derivative scalar Hessian;
- an uncompensated pointwise multiplier that enforces the normalized curvature readout produces an indefinite scalar kinetic block;
- regular positive-definite auxiliaries do not repair that block without a parent-derived companion term;
- the $R = 0$ normalization apex is non-Fréchet differentiable; and
- the unsuppressed cosmological scalar-memory realization exhibits a finite-k Floquet instability.

The Floquet obstruction has since been closed at the effective-theory level: the activation-fork resolution shows that on the conservative total-rate branch the minimal admissible low-rate activation susceptibility is quadratic, $g_2(d) = d^2/(d^2 + \mathcal{D}_X^2)$, pointwise stable through scalar-oscillation onset in the deterministic weak-coupling baseline, with a minimum margin above 4×10^3 while retaining order-one response at the late-inspiral thresholds. The action-level Dirac/Ward variation of that activated map remains outstanding.

The surviving construction is therefore not the minimal reduced local action. It is a parent-level split-leg open-system completion with:

$$Q = P_A \mathcal{C}^A$$

defined before reduction, a first-order closure channel, a preserved lapse–shift–clock constraint rank, an explicit Ward-completing environment, and a nontrivial physical readout.

11.3 Why the present test still matters

The null theorem is valid whether the final STF completion succeeds or fails. If a healthy parent theory produces universal coupling only, it predicts closure. If it produces a differential tensor leg, the residual template measures that leg. If no healthy parent channel exists, the STF mapping is discarded but the observable remains a useful generic test of differential propagation.

This makes clock closure a good interface between theory construction and data analysis: the observable can be defined now, while coefficient priors can be updated as the parent theory matures.

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12. Falsification and interpretation matrix

OBSERVATION	UNIVERSAL-LAPSE SECTOR	DIFFERENTIAL STF PROPAGATION BRANC
$\mathcal{R}_{2\mathrm{C}} = 0$, no periodicity	passes	unconstrained if below sensitivity
$\mathcal{R}_{2\mathrm{C}} \neq 0$, no common period	fails as sole explanation	not the minimal coherent-scalar template
common-period residual with endpoint/redshift scaling	fails as sole explanation	qualitatively supported
same periodic drift in EM and	closure may still hold	differential branch disfavored

OBSERVATION	UNIVERSAL-LAPSE SECTOR	DIFFERENTIAL STF PROPAGATION BRANC
GW channels		
birefringent periodic residual	fails as sole explanation	parity-even branch disfavored
detectable residual at the V8.1 period but amplitude $\gg 10^{-37}\,\mathrm{s}^{-1}$	fails	V8.1 coefficient benchmark fails

A nonzero residual is not automatically evidence for STF. The distinctive evidence would be the joint pattern: nonclosure, common internal period, endpoint structure, correct redshift scaling, and compatibility with independent tensor-sector constraints.

13. Limitations

13.1 Coherence assumptions

The source and observer scalar phases need not be coherent across cosmological distances. The phase variables should be marginalized independently unless a concrete scalar halo or cosmological coherence model is supplied. The phase-independent envelope remains valid without coherence.

13.2 Source dynamics

A scalar–Gauss–Bonnet coupling can also affect binary generation. The present paper isolates propagation. A full waveform analysis must include or bound generation-side corrections so that they are not absorbed into $\mathcal{R}_{2\mathrm{C}}$.

13.3 Amplitude propagation

Time-dependent tensor kinetic normalization also modifies the amplitude. We use the phase/redshift channel because it maps directly onto clock closure. A joint amplitude–phase analysis may improve discrimination but requires a careful luminosity-distance and selection model.

13.4 Environmental acceleration

Long-lived binaries may experience acceleration in host galaxies, clusters, triples, or nuclear environments. These effects can generate the same $f^{-13/3}$ scaling as cosmological redshift drift. Cross-messenger host characterization and population marginalization are mandatory.

13.5 Detector timing

The null test compares clocks. Detector timing calibration, barycentric corrections, and long-term ephemeris errors therefore require exceptional control. A periodic observer endpoint term can be especially vulnerable to a common timing systematic.

13.6 Parent theory

The metric coefficient $F_{,\phi}$, the closure support coefficient κ, and any cosmological activation factor must be derived from one explicit healthy parent completion. The current response matching is a conditional coefficient map, not yet such a derivation.

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14. Next decisive calculations

The paper is sufficient to define the test, prove its null theorem, and provide a conditional STF template. It is not the end of the theory program. The next calculations should be performed in this order:

1. **Complete the parent channel.** Exhibit a nontrivial $Q = P_A \mathcal{C}^A$ whose Poisson brackets preserve the lapse–shift–clock constraint rank and whose environment completes the Ward identity.
2. **Derive the tensor coefficient.** Obtain $F_{,\phi}$, including any activation and memory factors, from that parent theory rather than response matching alone.
3. **Derive the activation susceptibility.** The activation fork is resolved at the effective level: the conservative total-rate branch selects a quadratic low-rate susceptibility, $g_2(d) = d^2/(d^2 + \mathcal{D}_X^2)$; the former square-root gate fails on the fast scalar-Ricci rate; and the earlier numerical identification of the $\sim 10^{-20}$ Floquet suppression with the capacity projection does not survive — any unification must be structural. What remains is action-level: derive the quadratic susceptibility, its threshold scale, and its channel eigenvalues from the parent theory while preserving the constraint rank and Ward identity.
4. **Construct a joint generation–propagation waveform.** Include source-side scalar corrections, propagation endpoints, amplitude transport, and the exact high-pass kernel.
5. **Forecast the null observable.** Use a decihertz population with realistic host accelerations, redshift uncertainties, detector timing, and an external EM redshift-drift prior.

Only after steps 1–3 can the tiny V8.1 benchmark be promoted from a conditional estimate to a prediction of a complete theory.

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15. Conclusion

The two-clock idea admits a clean operational test.

For any cosmology in which photons, gravitational waves, and matter clocks share one physical metric, an arbitrary universal lapse cancels from the comparison of GW-inferred and EM-measured redshift drift. The exact null prediction is

$$\mathcal{R}_{2\mathrm{C}} = 0.$$

A weak differential tensor speed violates closure according to

$$\mathcal{R}_{2\mathrm{C}} = \frac{1}{2}\left[\frac{\dot{\delta c}_{T,e}}{1+z} - \dot{\delta c}_{T,o}\right].$$

An oscillating scalar–Gauss–Bonnet branch converts the STF internal clock into a periodic endpoint phase while the standard acceleration phase continues to track the universal clock. This is the precise sense in which the two clocks become separately observable.

The present STF V8.1 coefficient benchmark is far too small for foreseeable detection: at $z = 1$, its maximum closure residual is about $2.60 \times 10^{-37}\,\mathrm{s}^{-1}$. That numerical result does not weaken the theorem. It clarifies the scientific role of the proposal. Clock closure is a model-independent null test; STF supplies one conditional, highly suppressed template for violating it.

The conceptual result is therefore stronger than a claimed signal:

> A universal clock can alter cosmological dynamics, but it cannot by itself make two messengers disagree. Cross-messenger nonclosure isolates genuinely differential propagation.

Appendix A. Endpoint expansion and sign check

Let

$$\delta c_T = \epsilon \cos\Theta, \qquad \dot\Theta = m_s.$$

The endpoint phase is

$$\delta\Psi_T = \frac{2\pi f}{m_s}\left[-\epsilon_o \sin(\Theta_{c,o} - m_s\Delta T) + (1+z)\epsilon_e \sin\left(\Theta_{c,e} - \frac{m_s\Delta T}{1+z}\right)\right].$$

Using

$$\sin(\Theta - x) = \sin\Theta - x\cos\Theta - \frac{x^2}{2}\sin\Theta + \mathcal{O}(x^3),$$

the quadratic term is

$$\delta\Psi_T^{(2)} = 2\pi f \Delta T^2 \frac{m_s}{2}\left[\epsilon_o \sin\Theta_{c,o} - \frac{\epsilon_e \sin\Theta_{c,e}}{1+z}\right].$$

But

$$\dot{\delta c}_T = -m_s \epsilon \sin\Theta,$$

so

$$\delta\Psi_T^{(2)} = 2\pi f \Delta T^2 \frac{1}{2}\left[\frac{\dot{\delta c}_{T,e}}{1+z} - \dot{\delta c}_{T,o}\right].$$

The endpoint and redshift derivations therefore agree exactly at this order.

Appendix B. Acceleration-phase coefficient

Starting from

$$\Psi_{\rm acc} = 2\pi f X \Delta T^2, \qquad \Delta T = \frac{5}{256}\mathcal{M}_z^{-5/3}(\pi f)^{-8/3},$$

we obtain

$$\Psi_{\rm acc} = 2\pi f X \frac{25}{65536} \mathcal{M}_z^{-10/3} (\pi f)^{-16/3}.$$

Since $f = (\pi f)/\pi$,

$$\Psi_{\rm acc} = \frac{25}{32768} X \mathcal{M}_z^{-10/3} (\pi f)^{-13/3}.$$

The companion code verifies the equality numerically to machine precision.

Appendix C. Convention ledger

ITEM	CONVENTION USED HERE
Metric signature	$(-+++)$
Proper time	$d\tau = L_U dT$
Hubble rate	$H = a^{-1} da/d\tau$
GW speed	$c_T = 1 + \delta c_T$
Closure residual	$X_{\rm GW} - X_{\rm EM}$
Fourier acceleration phase	$+2\pi f X \Delta T^2$
sGB sign	carried by $\sigma_{\rm GB}$
Scalar frequency	$\dot{\Theta}_I = m_s$, with m_s converted from eV using $\hbar$

Appendix D. Reproducibility

Run:

```
python stf_cross_messenger_clock_closure_checks.py
```

The script checks:

1. exact cancellation for arbitrary endpoint lapse values;
2. equality of the direct and closed forms of the corrected acceleration phase;
3. agreement between the endpoint expansion and the redshift-derivative residual;
4. the $z = 1$ ΛCDM benchmark;
5. the conversion of $m_s = 3.94 \times 10^{-23}$ eV to a 3.326-year period; and
6. the inspiral and endpoint phase values in Table 1.

No nonstandard Python packages are required.

References

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Result ledger

Established in this paper

- exact universal-lapse cancellation;
- corrected positive acceleration-phase coefficient;
- corrected emitter-minus-observer differential-speed residual;
- equivalence of redshift-drift and endpoint-phase derivations;
- conditional scalar–Gauss–Bonnet STF template;

- numerical V8.1 closure benchmark.

Not established in this paper

- a complete covariant STF parent action;
- derivation of the response-matched metric coefficient from the capacity coefficient;
- equality of the cosmological activation and capacity-projection operators;
- detectability of the V8.1 propagation signal;
- a full generation-plus-propagation waveform analysis.

CITATION

```
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  author = {Paz, Z.},
  title = {Cross-Messenger Clock Closure in Two-Clock Cosmology},
  year = {2026},
  version = {V1.0},
  url = {https://existshappens.com/papers/clock-closure/}
}
```

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