# D3 Charge-Frame Selector No-Go Record and Conditional Parent

A universal-lapse null theorem and an STF propagation template

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# STF D3 Charge-Frame Selector No-Go Record and Conditional Parent

## V2.0 standalone consolidation: from the reciprocal channel theorem to the spherical-wall $SU(2)_R$ frontier

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## Supersession statement

This paper supersedes *STF D3 Charge-Frame Selector No-Go Record and Conditional Fork*, V1.0. V1.0 remains part of the audit history, but its stopping point is no longer current. The present version incorporates the subsequent real-structure, inverse-$S$ Hopf-cup, local-sky globalization, Euler-obstruction, and spherical compensator calculations.

No earlier correction is silently absorbed. The correction record is preserved in Section 16, and every route that was considered but not tested is kept separate from the proved obstructions in Section 15.

## Abstract

The STF activation line needs an ordered map from two closure primitives to the electric–magnetic charge lattice of a D3 gauge sector. This paper consolidates the complete selector audit through its current frontier.

The invariant representation result is exact. For the positive determinant-one D3 duality metric

$$\mathcal{M}(\tau) = \frac{1}{\tau_2}\begin{pmatrix} |\tau|^2 & \tau_1 \\ \tau_1 & 1 \end{pmatrix},$$

the symmetric square of its positive square root has reciprocal spectrum

$$\operatorname{spec}\operatorname{Sym}^2\sqrt{\mathcal{M}} = \{s^2, 1, s^{-2}\}.$$

At the separately assumed CP-even benchmark

$$\tau_{\text{vis}} = i\alpha^{-1},$$

the unordered susceptibility set is

$$\boxed{\{\alpha^{-1}, 1, \alpha\}},$$

and the inverse-threshold set is

$$\boxed{\mathcal{D}_1\{\alpha, 1, \alpha^{-1}\}}.$$

Neither set orders the outer weights on the closure monomials $(RR, RA, AA)$. An integral map

$$K = (\boldsymbol{n}_R\, \boldsymbol{n}_A) \in SL(2,\mathbb{Z})$$

is still required. The balanced quadratic and exact BPS endpoint functionals have precisely four minima,

$$\{I, -I, J, -J\}.$$

The endpoint-orientation audit corrected the physical charge order. In the fixed doublet $(F_D, F)$, the first coordinate is D1/magnetic and the second is F1/electric. Positive advanced-electric orientation gives $K = I$, while positive retarded-electric orientation gives

$$K = -J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

These frames have the same unordered spectrum and opposite outer ordered weights.

The compactification and causal selector searches did not choose between them. The published CICY #7447/$\mathbb{Z}_{10}$ flavour construction is a heterotic bundle model rather than a direct Type-IIB D3 parent; its smooth rank-one quotient has no genus-one fibration; and its exact ambient-linear quotient normalisers have no nondegenerate $\Omega$-odd O3/O7 branch. An independent global orientifolded $dP_5$ model supplies a genuine compact D3 sector and primitive visible electric ray, but no retarded/advanced label. Standard Schwinger–Keldysh doubling factorizes from the charge lattice and cannot identify contour position with F1/D1 species. A duality wall implements a specified modular map but does not choose it. Homogeneous same-coupling transparency initially appeared to conditionally select $I$.

The real-structure audit closes that apparent easy branch. Time reversal exchanges the primitive closure classes,

$$C_{\text{cl}} = \pm X, \qquad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

whereas ordinary D3 electromagnetic time reversal is split,

$$C_T = \mathrm{diag}(-1, 1).$$

Every integral intertwiner has even determinant. The minimal index-two image excludes the primitive electric vector already present in the $dP_5$ spectrum and mixes the ordered $(RR, RA, AA)$ channels. Preserving the primitive ordered closure basis therefore requires an electromagnetic $S$ factor. The preferred remaining frame is the inverse- $S$ map $K = -J$, with

$$\tau_{\mathrm{cl}} = -1/\tau_{\mathrm{vis}} = i\alpha.$$

A direct linear differential-cohomology map from a closure winding to a wall connection fails by degree or by circularly selecting one fiber circle. The bilinear normalized Hopf class

$$u_\gamma = \frac{\omega_R \wedge \omega_A}{4\pi^2}, \qquad \int_{T^2_\gamma} u_\gamma = 1,$$

does something different and useful: it supplies the primitive level of a two-field, CP-even, unimodular BF wall. In the restricted minimal wall class the matrix is unique up to orientation, and the corrected endpoint orientation yields $K = -J$. On a fixed trivialized patch the wall has primary Dirac rank four, zero local degrees of freedom, zero stress, and zero positive-semidefinite noise. It exactly reproduces the ordered susceptibility and threshold triples

$$(\alpha, 1, \alpha^{-1}), \qquad (\alpha^{-1}, 1, \alpha).$$

Globalization forces two further corrections. First, a nonzero null vector cannot be orthogonal to a timelike vector. The published $g(k, U) = 0$ time-orientation equator is therefore empty on a normalized observer sky. A cooriented timelike physical wall repairs the local loop through its spacelike normal $n$, using $g(k, n) = 0$. Second, the family of Clifford tori over the wall is

$$E_\gamma = S(L) \times_W S(L^{-1}), \qquad 2c_1(L) = e(P),$$

with Chern vector $(\ell, -\ell)$. The primitive fiber-area class globalizes if and only if $\ell = 0$. For a spherical wall, $\ell = 1$, so the naive compact spherical inverse- $S$ parent fails.

The obstruction has a unique primitive diagonal cancellation. A degree-one $SU(2)_R$ Hopf texture supplies eigenlines $M_\pm$ with Chern numbers $\pm 1$, and the assignment

$$R : L \otimes M_-, \qquad A : L^{-1} \otimes M_+$$

trivializes both phase lines without mixing the ordered pair. This is a global bundle-level pass and preserves the level-one inverse- $S$ response. It is not yet a physical parent: the closure pair has not been derived as an $SU(2)_R$ doublet with the required opposite charges; a dynamical texture adds energy and modes; an independent background leaves an $F_{\mathcal{B}} J_{\mathcal{B}}$ Ward force; and the composite normal-bundle action, full Hessian, anomaly inflow, dual-coupling region, and enlarged noise kernel remain open.

The exact current status is therefore

| | |
|---|---|
| unordered reciprocal representation: | **PASS**, |
| audited direct/static/SK selectors: | **NO SELECTION ORFAIL**, |
| primitive transparent same-coupling branch: | **FAIL**, |
| local Hopf-cup inverse-$S$ parent: | **CONDITIONALPASS**, |
| naive spherical globalization: | **FAIL**, |
| $SU(2)_R$ compensation at bundle level: | **PASS**, |
| full Ward/Dirac/anomaly/UV parent: | **OPEN**, |
| unconditional D3 charge-frame go: | **NOT ESTABLISHED**. |

This is a complete no-go record for the mechanisms actually audited. It is not a framework-wide no-go and does not convert unattempted routes into failures.

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## 1. Editorial architecture

This V2.0 synthesis makes six explicit editorial choices.

### Choice 1 — invariants before labels

The unordered reciprocal spectrum is proved before any retarded/advanced assignment. No eigenvalue set is used as evidence for an ordered channel map.

### Choice 2 — one fixed charge convention

Throughout,

$$\boldsymbol{\mathcal{F}} = \begin{pmatrix} F_D \\ F \end{pmatrix}, \qquad \boldsymbol{n} = \begin{pmatrix} q_D \\ p_F \end{pmatrix},$$

with D1/magnetic first and F1/electric second. The convention is never relabeled to make a candidate diagonal.

### Choice 3 — corrections remain visible

The CICY identity, orientifold descent, endpoint orientation, isolation gap, coupling premise, causal real structure, and sky-equator correction are recorded as corrections. Later theorems narrow earlier conclusions rather than rewriting their history.

### Choice 4 — local, global-bundle, and physical-parent passes are distinct

A local BF constraint count, a global bundle trivialization, and a Ward-closed UV action are three different gates. Passing one does not upgrade the others.

### Choice 5 — obstruction and non-attempt are separate ledgers

Only a tested construction can appear in the no-go record. Unconstructed gerbe kernels, nonambient involutions, alternate brane defects, and related routes remain explicitly unexplored.

### Choice 6 — V2.0 ends at the current action frontier

The paper does not present the explicit composite $SU(2)_R$-twisted D3 action as though it existed. It ends at the representation/Ward/UV task that would decide the remaining route.

These are editorial decisions, not new physical assumptions.

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## 2. Fixed notation and the two-layer target

Let

$$e_D = \begin{pmatrix}1\\0\end{pmatrix}, \qquad e_F = \begin{pmatrix}0\\1\end{pmatrix}, \qquad J = \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}.$$

The closure basis is

$$\boldsymbol{\omega} = \begin{pmatrix}\widehat{\omega}_R\\ \widehat{\omega}_A\end{pmatrix},$$

and the endpoint map is written by columns,

$$K = (\boldsymbol{n}_R\ \boldsymbol{n}_A).$$

The audit separates:

1. the **spectral problem**—the unordered response eigenvalues; and
2. the **selector problem**—which closure primitive maps to which D3 charge and hence which ordered $(RR, RA, AA)$ weight.

The first problem is solved conditionally on a D3 sector and benchmark coupling. The second is the subject of the no-go record.

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## 3. The D3 channel theorem

The FP V8.1 metric-only block-diagonal reduction does not itself contain a D3 photon or three-channel matrix. The D3 sector is a separate ultraviolet candidate.

For $\tau = \tau_1 + i\tau_2$, define

$$\mathcal{M}(\tau) = \frac{1}{\tau_2}\begin{pmatrix}\tau_1^2 + \tau_2^2 & \tau_1\\ \tau_1 & 1\end{pmatrix}.$$

It is positive and has determinant one. If the eigenvalues of $\sqrt{\mathcal{M}}$ are $(s, s^{-1})$, then

$$\operatorname{spec}\operatorname{Sym}^2\sqrt{\mathcal{M}} = \{s^2, 1, s^{-2}\}.$$

At $\tau = i\alpha^{-1}$,

$$\mathcal{M}_0 = \operatorname{diag}(\alpha^{-1}, \alpha),$$

so the unordered susceptibility and threshold sets are

$$\{\alpha^{-1}, 1, \alpha\}, \qquad \mathcal{D}_1\{\alpha, 1, \alpha^{-1}\}.$$

This is a representation theorem, not an ordered selector. The primitive shear

$$K_T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

is integral, symplectic and unimodular, yet changes the fixed-basis numerical extremes. Hence

$$K \in SL(2,\mathbb{Z}) \quad \not\Rightarrow \quad \text{the ordered } \alpha\text{-ladder}.$$

**Status:** unordered theorem **PASS**; ordered assignment **OPEN at this stage**.

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## 4. Minimum tension and the fourfold degeneracy

The normalized closure cup pairing fixes the antisymmetric form but not a positive energy norm. On the Clifford torus the Hodge Gram matrix is $G_{\mathrm{Cliff}} = I$, but identifying it with a D3 endpoint action is a separate hypothesis.

For the candidate positive functionals

$$E_{\mathrm{flux}}(K) = C\,\mathrm{Tr}(K^T \mathcal{M}_0 K),$$

and

$$E_{\mathrm{BPS}}(K) = LT_0 \sum_{I=R,A} \sqrt{\boldsymbol{n}_I^T \mathcal{M}_0 \boldsymbol{n}_I},$$

the integral minimization is exact. Writing

$$K = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z}),$$

the quadratic cost obeys

$$E_{\mathrm{flux}}/C = \alpha^{-1}(a^2+b^2) + \alpha(c^2+d^2) \geq \alpha^{-1} + \alpha.$$

Equality forces unit integer rows and leaves exactly

$$\boxed{K \in \{I, -I, J, -J\}}.$$

The exact BPS functional has the same four minima. Thus balanced positive energy reduces the frame ambiguity but does not orient it.

The corrected nearest gaps are

$$\Delta E_{\mathrm{flux}}/C = \alpha,$$

and

$$\frac{\Delta E_{\mathrm{BPS}}}{LT_0} = \sqrt{\alpha^{-1}+\alpha} - \sqrt{\alpha^{-1}} = 3.11682355 \times 10^{-4}.$$

**Status:** fourfold minimum **THEOREM for the stated actions**; use as an STF endpoint action **CONDITIONAL**.

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## 5. Endpoint orientation and the ordered fork

The gauge-invariant open $(p, q)$-string/D3 endpoint module fixes charge integrality and boundary gauge cancellation. Twisted self-duality leaves one Maxwell photon, not two independent photons.

The first major self-correction is physical: in the fixed $(F_D, F)$ doublet,

$$e_D = (1, 0)^T \text{ is D1/magnetic}, \qquad e_F = (0, 1)^T \text{ is F1/electric}.$$

Therefore

$$\boldsymbol{n}_A = e_F \Longrightarrow K = I,$$

while

$$\boldsymbol{n}_R = e_F \Longrightarrow K = -J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

The resulting ordered data are

| FRAME | ELECTRIC CLOSURE LEG | ORDERED SUSCEPTIBILITIES $(RR, RA, AA)$ | ORDERED THRESHOL |
|---|---|---|---|
| $K = I$ | $A$ | $(\alpha^{-1}, 1, \alpha)$ | $(\alpha, 1, \alpha^{-1})$ |
| $K = -J$ | $R$ | $(\alpha, 1, \alpha^{-1})$ | $(\alpha^{-1}, 1, \alpha)$ |

The unordered sets agree. The ordered assignments do not.

**Status:** exact two-way fork after positive orientation; no selector yet.

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## 6. Compactification audits

### 6.1 CICY #7447 and the wrong string corner

The exact geometry is

$$X_{7447} = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \end{array}\right]^{5,45},$$

with quotient data

$$(h^{1,1}, h^{2,1})(X/\mathbb{Z}_{10}) = (1, 5), \qquad \widehat{H}^3 = 12.$$

The three published STF flavour papers use a heterotic $\mathbb{Z}_{10}$-equivariant $SU(4)$ monad bundle and bundle cohomology. Those data are real compactification data, but they do not directly define an O3/O7 projection, D3 Chan–Paton sector, F1/D1 endpoint lattice, or D3 axio-dilaton.

The standard heterotic/F-theory shortcut also fails on the smooth quotient. It has Picard rank one, every divisor is $D = a\widehat{H}$, and

$$D^3 = 12a^3.$$

A nonzero genus-one-fibration divisor would require $D^3 = 0$, which is impossible. The rank-four period lattice and exact monodromies improve the input but do not canonically select a rank-two D3 charge plane.

**Status:** direct reuse **FAIL**; standard geometric translation **OBSTRUCTED**; nongeometric or birational duals **UNEXPLORED**.

### 6.2 The $\mathbb{Z}_{10}$ O3/O7 equivariance obstruction

The first feasibility pass found nonempty quotient-compatible polynomial spaces and therefore assigned a conditional grade. The later residue- character audit corrected it. For the exact ambient-linear normalisers,

$$F(h) = \frac{\det h \ \det \rho_h}{\det \pi_h} = +1,$$

where O3/O7 descent requires $F(h) = -1$. Exhaustive enumeration of

$$26 \times 2^5 \times 2^5 = 26{,}624$$

signed-monomial candidates leaves 24 compatible normaliser representatives and zero nondegenerate $\Omega$ -odd branches.

The known O3/O7 data of the #7447 cover use a polynomial parity incompatible with the exact quotient equation representation. Cover fixed-locus numbers therefore do not descend to this quotient branch.

**Status:** ambient-linear #7447/$\mathbb{Z}_{10}$ O3/O7 route **OBSTRUCTED**. Nonambient/birational involutions are not covered.

### 6.3 The independent global $dP_5$ parent

The orientifolded $dP_5$ construction supplies a genuine compact D3 quiver. In the displayed zero-flux charge completion,

$$-48 + 8 + 20 \times 2 = 0.$$

The family hypercharges, multiplied by six, are

$$(1, -3, 4, -2, 0, -6),$$

with greatest common divisor one. Thus the visible spectrum contains a primitive electric ray.

The compactification nevertheless contains no retarded/advanced column label. Both $I$ and $-J$ use the same electric ray and unordered spectrum.

> **Static compactification no-selection theorem for the audited data class.** Tadpoles, intersection data, brane nodes, Chan–Paton charges, and a visible electric lattice do not select the STF causal column without an additional oriented closure-to-endpoint map.

The source also does not derive $\tau = i\alpha^{-1}$ after flux stabilization, Stückelberg reduction, mixing, thresholds and running.

**Status:** compact D3/electric parent **PASS**; causal selector **NO SELECTION**.

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## 7. Schwinger–Keldysh charge factorization

For the branch endpoint source,

$$\boldsymbol{J}_+^T \boldsymbol{A}_+ - \boldsymbol{J}_-^T \boldsymbol{A}_- = \boldsymbol{J}_r^T \boldsymbol{A}_a + \boldsymbol{J}_a^T \boldsymbol{A}_r.$$

The SK transformation acts on the contour factor, while electromagnetic duality acts on charge:

$$R_{\mathrm{SK}} \otimes I_2, \qquad I_{\mathrm{SK}} \otimes K.$$

They commute, and

$$\mathcal{V}_{\mathrm{parent}} = V_{\mathrm{SK}} \otimes \Gamma_{(q_D, p_F)}.$$

Every fixed charge has both contour copies. The branch-to-$r/a$ change contains halves and is not an integral charge-frame map. Assigning F1 and D1 to opposite branches also leaves a nonzero source on the equal-field locus and violates $Z[A, A] = 1$.

Twisted self-duality and terminal SK gluing commute and return one Maxwell field, but do not correlate charge and contour labels.

**Status:** standard factorized SK–D3 parent **PASS**; SK selection of $K$ **FAIL**. Nonfactorizing defects remain unexplored.

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## 8. Duality walls and the V1.0 coupling fork

An Abelian $S$-wall contains

$$I_S[A, \widehat{A}] = \frac{i}{2\pi} \int_W A \wedge d\widehat{A}$$

in Euclidean signature. Stacked generator walls implement any specified $SL(2, \mathbb{Z})$ label. The wall implements its label; it does not dynamically choose it.

In the corrected endpoint convention, $K = I$ is transparent and $K = -J$ is inverse-$S$. A topological $S$-wall requires

$$\hat{\tau} = -1/\tau.$$

For $\tau = iy$, $y > 1$, the fixed-point equation gives

$$\mathrm{Stab}_{SL(2,\mathbb{Z})}(iy) = \{\pm I\}.$$

V1.0 therefore recorded a conditional same-coupling theorem: if the two sides are the same homogeneous fixed-coupling theory, there is no localized source, and positive orientation excludes $-I$, then $K = I$.

The displayed $SO(8)$ D7/O7 stack does not supply the alternative wall: in the standard weak-coupling monodromy convention of Sen, four coincident D7 branes on an $O7^-$ plane give $T^4(-T^{-4}) = -I$, so its net monodromy is $-I$, not $-J$.

This was the exact V1.0 fork. Section 9 explains why it is no longer the current endpoint.

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## 9. Integral real structure closes the transparent primitive branch

Time-orientation reversal exchanges the Hopf/anti-Hopf primitives. Up to the linear versus anti-linear sign convention,

$$C_{\mathrm{cl}} = \pm X, \qquad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Ordinary D3 time reversal makes magnetic charge odd and electric charge even:

$$C_T = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$$

For the plus convention, an integral intertwiner satisfies

$$KX = C_T K \quad \Longrightarrow \quad K = \begin{pmatrix} a & -a \\ c & c \end{pmatrix}, \qquad \det K = 2ac.$$

Every determinant is even. No $GL(2,\mathbb{Z})$ equivalence exists. The same conclusion holds for $-X$.

The minimal image is the index-two lattice

$$L_2 = \{(q_D, p_F) : q_D \equiv p_F \pmod 2\},$$

generated by $(1,1)$ and $(-1,1)$. Its smallest pure-electric vector is $(0,2)$, so it excludes the primitive electric charge already present in the $dP_5$ spectrum.

After determinant normalization the index-two pullback preserves the unordered reciprocal spectrum, but its eigenvectors are $R - A$ and $R + A$. It mixes $RR, RA, AA$ and does not preserve the published ordered assignment.

A $\mathbb{Z}_2$ saturation adjoining $(R \pm A)/2$ diagonalizes the involution at one coupling, but changes the primitive causal basis and requires a revised channel claim. It is not the original construction.

The four endpoint minima instead induce the $S$-twisted operations

$$C_T S = X, \qquad SC_T = -X, \qquad S = -J.$$

Thus preserving the primitive ordered R/A basis and ordinary D3 physics requires an electromagnetic $S$ factor. At the benchmark,

$$i\alpha^{-1} \xrightarrow{S} i\alpha.$$

**Updated status:** primitive transparent same-coupling interpretation **FAILS** integral real-structure equivariance. The inverse- $S$ route is the preferred minimal ordered route. The $\mathbb{Z}_2$-saturated same-coupling route is a conditional alternative only after revising the ordered-channel statement.

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## 10. Differential-cohomology attachment: linear failure and bilinear pass

### 10.1 Linear degree obstruction

A winding class has differential degree one and a Poincare line kernel has degree two. Full $T^2$ pushforward gives

$$1+2-2=1,$$

not the degree two of a U(1) connection. One-circle pushforward gives the right degree,

$$1+2-1=2,$$

but requires choosing R or A and therefore inserts the desired selector. Using the full area pair with a line kernel is degree-correct but collapses the rank-two input to $H^2(T^2,\mathbb{Z}) \cong \mathbb{Z}$.

> **Linear attachment theorem.** Within the stated line-kernel class, no full-torus linear pushforward both lands in U(1) gauge data and retains two independent causal generators.

This theorem does not cover gerbe kernels, higher-form carriers, extra circle fields, or nonlinear parents.

### 10.2 Oriented Hopf-cup level

Define

$$u_\gamma = \frac{\omega_R \wedge \omega_A}{4\pi^2}, \qquad \int_{T^2_\gamma} u_\gamma = 1.$$

Assume a closed oriented wall $W$, a torus bundle $\pi : E_\gamma \to W$, and a differential refinement with $\pi_* \widehat{u}_\gamma = 1$. Then

$$\exp\left[2\pi i \int_{E_\gamma} \widehat{u}_\gamma \smile \pi^* \widehat{A}_{\mathrm{cl}} \smile \pi^* \widehat{A}_{\mathrm{vis}}\right]$$

reduces to the level-one BF wall

$$\exp\left[\frac{i}{2\pi} \int_W A_{\mathrm{cl}} \wedge dA_{\mathrm{vis}}\right].$$

The complete cup product supplies an oriented integer level rather than an individual charge column.

### 10.3 Restricted wall uniqueness and orientation

For two U(1) restrictions, no added propagating wall field, CP-even absence of diagonal Chern–Simons levels, and a nondegenerate unimodular first-order wall,

$$\mathsf{K} = \begin{pmatrix} 0 & k \\ k & 0 \end{pmatrix}, \qquad |\det \mathsf{K}| = k^2 = 1.$$

Therefore $k = \pm 1$. The canonical order $R \wedge A$, together with the corrected closure-to-visible endpoint orientation, gives

$$K = -J, \qquad R \mapsto e_F, \qquad A \mapsto -e_D.$$

This is a conditional uniqueness theorem for the restricted wall class, not for all possible interfaces.

### 10.4 Local Dirac, Ward and noise audit

The BF primary matrix is

$$\Omega = \frac{1}{2\pi} \mathsf{K}_{BF} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \operatorname{rank} \Omega = 4.$$

Four second-class spatial constraints and four first-class temporal/Gauss constraints leave zero local wall degrees of freedom. The metric-independent wall has zero stress. Bulk stress matches when

$$\tau_{\text{cl}} = -1/\tau_{\text{vis}},$$

including spatial Ward components on a closed smooth wall. Duplicating the same wall on both SK copies preserves $Z[A, A] = 1$. The wall contributes

$$N_W = 0 \succeq 0.$$

The exact ordered response is

$$(\chi_{RR}, \chi_{RA}, \chi_{AA}) = (\alpha, 1, \alpha^{-1}),$$

with thresholds

$$(\mathcal{D}_{RR}, \mathcal{D}_{RA}, \mathcal{D}_{AA})/\mathcal{D}_1 = (\alpha^{-1}, 1, \alpha).$$

**Status:** local fixed-sector inverse-$S$ parent **CONDITIONAL PASS**; global bundle and threshold topology change not yet included.

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## 11. The local-sky correction and wall-normal repair

For a unit timelike vector $U$, decompose a null vector as

$$k = aU + s, \qquad g(s, U) = 0.$$

Nullness gives $|s|^2 = a^2$, while $g(k, U) = -a$. Hence a nonzero null vector cannot satisfy $g(k, U) = 0$.

The published expression

$$\gamma_U = \{[k] : g(k, U) = 0\}$$

does not define an equator. On the future normalized sky, $g(k,U)=-1$ everywhere. On the real projective cone the sign is not well-defined. Time orientation selects a cone component, not a hemisphere or a great circle within one sky.

The local Hopf theorem survives for any independently supplied loop $\gamma\subset S^2$. What fails is time-orientation-only canonicity.

A timelike, oriented and cooriented wall supplies a spacelike unit normal $n$. With $U$ tangent to the wall,

$$P=(\mathrm{span}\{U,n\})^{\perp}$$

is an oriented spacelike plane and

$$\gamma_{U,n}=\{k=U+s:|s|=1,\ g(k,n)=0\}=S(P)$$

is a genuine equator. Its normal-crossing sign is not fixed by future support; identifying its sides as R/A requires an additional directed propagator or endpoint condition.

**Status:** published time-only equator **FAIL**; timelike wall-normal loop **PASS**; global R/A side assignment **CONDITIONAL**.

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## 12. Globalization and the Euler obstruction

Assume the ambient parent is spin. A spin lift of $P$ supplies a line $L$ with

$$2c_1(L)=e(P),\qquad \ell=c_1(L).$$

The family of Clifford tori is

$$E_\gamma=S(L)\times_W S(L^{-1}),$$

with Chern vector $(\ell,-\ell)$. For primitive fiber classes $x_R,x_A$,

$$d_2x_R=\ell,\qquad d_2x_A=-\ell,$$

and multiplicativity gives

$$\boxed{d_2(x_Rx_A)=\ell(x_R+x_A)}.$$

Therefore an integral class restricting to the primitive area generator on each fiber exists if and only if

$$\boxed{\ell=0\in H^2(W;\mathbb{Z})}.$$

Fiber orientation alone is insufficient. Nonzero torsion $\ell$ also obstructs the integral/differential class even when de Rham curvature misses it.

For $W\simeq\mathbb{R}\times\Sigma_g$,

$$\int_{\Sigma_g}\ell=1-g.$$

Thus local contractible patches and toroidal walls pass. The spherical wall has $\ell = 1$ and fails; compact higher-genus walls also fail.

This is a global patching obstruction, not a local rank loss. The local BF rank, response and zero-noise checks remain valid in trivializing patches.

**Status:** naive compact spherical Hopf-cup inverse-$S$ parent **FAIL**.

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## 13. The compensator audit

### 13.1 Unique primitive cancellation

Let a line $\mathcal{B}$ act with charges $(+1, -1)$ on $(R, A)$, and write $b = c_1(\mathcal{B})$. The effective Chern vector is

$$(\ell + b, -\ell - b).$$

Both entries vanish if and only if

$$\boxed{b = -\ell}.$$

On the sphere the required line has $c_1 = -1$. Higher primitive charge magnitude cannot solve $qb = -1$ integrally. The compensator is unique up to simultaneous orientation reversal.

### 13.2 Candidates that fail the minimal test

- The ordinary wall normal is trivial because coorientation supplies a global nonzero section. It has $c_1 = 0$.
- A homogeneous or radial axio-dilaton has zero tangential duality curvature. A smooth single-valued $\tau : S^2 \to \mathbb{H}$ also integrates to zero.
- A half-charged pure duality line would require $c_1(L_{\mathcal{D}}) = -2$, hence angular modular monodromy, chiral defects and an unproved half-charge assignment.
- A unit D3 magnetic monopole has the right Chern number but is a D1 endpoint. It inserts a charge primitive and has positive energy $E_B = \pi m^2/(2g^2 r^2)$. It fails the neutral zero-stress minimal compensator test.

The last two possibilities remain different, nonminimal constructions if their defects and sources are included; they are not declared impossible.

### 13.3 Explicit $SU(2)_R$ Hopf twist

Choose an $SU(2)_R \subset SU(4)_R$ doublet and a degree-one map

$$\widehat{n}_R : S^2_{\text{wall}} \to S^2_R.$$

The projectors

$$P_\pm = \frac{1}{2}(1 \pm \widehat{n}_R \cdot \sigma)$$

define complementary eigenlines $M_\pm$ with

$$c_1(M_+) = +1, \qquad c_1(M_-) = -1.$$

Assign

$$R : L \otimes M_-, \qquad A : L^{-1} \otimes M_+.$$

Both effective Chern classes vanish. The effective torus bundle is topologically trivial, the primitive global area class exists, the ordered pair is not mixed, and the inverse-$S$ level and response are unchanged.

**Status:** global bundle cancellation **THEOREM GIVEN THE ASSIGNMENT**; physical closure-to-$SU(2)_R$ assignment **OPEN**.

### 13.4 Physical cost and Ward qualification

If the texture is dynamical, the degree-one hedgehog has

$$E_R = 4\pi f_R^2$$

and adds wall modes and stress. If $\mathcal{B}$ is a prescribed external background, the on-shell identity is

$$\nabla_\mu T^\mu{}_\nu = F^{\mathcal{B}}_{\nu\mu} J^\mu_{\mathcal{B}},$$

which need not vanish in spatial components. If instead $\mathcal{B} = -\omega_L[g, U, n]$ is a natural composite twist in a complete diffeomorphism-invariant action, the improved stress is conserved classically on all field and embedding equations. That action and its implicit metric variation have not yet been constructed.

For a prescribed static background identical on both SK copies, the BF Dirac rank stays four and the added noise is zero. These facts do not prove the rank or positivity of a dynamical/composite completion.

**Status:** fixed-background BF rank/noise **PASS**; independent-background Ward closure **FAIL generically**; composite classical Ward closure **CONDITIONAL**; dynamical rank, anomaly inflow and full noise **OPEN**.

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## 14. Cumulative selector no-go record

The word **no-go** in this table applies only to the stated mechanism and hypotheses.

| CANDIDATE | TESTED PROPOSITION | RESULT | SCOPE |
|---|---|---|---|
| Integral symplectic pairing | $K \in SL(2,\mathbb{Z})$ selects the frame | **NO SELECTION** | Primitive shear counterexample |
| Clifford Hodge geometry | Geometric $G = I$ alone supplies a dynamical selector | **NO SELECTION** | Norm is not a derived endpoint action |
| Balanced BPS/flux energy | Positive endpoint energy selects one orientation | **FOURFOLD DEGENERACY** | Exactly $\{\pm I, \pm J\}$ for the stated actions |
| Positive electric primitive | Its closure leg is already determined | **ORIENTATION FORK** | $A$-electric $I$; $R$-electric $-J$ |
| Existing CICY flavour parent | Heterotic bundle data are direct D3 input | **FAIL** | Wrong corner for direct reuse |

| CANDIDATE | TESTED PROPOSITION | RESULT | SCOPE |
|---|---|---|---|
| Standard heterotic/F-theory shortcut | Smooth quotient is genus-one fibered | **OBSTRUCTED** | Picard-rank-one cubic |
| Ambient #7447/$\mathbb{Z}_{10}$ O3/O7 | Quotient normaliser is $\Omega$ -odd | **OBSTRUCTED** | Exhaustive ambient-linear class |
| Rank-four period lattice | Periods select the D3 rank-two plane | **NO SELECTION** | Multiple primitive invariant planes |
| Global $dP_5$ data | Static brane data label R/A | **NO SELECTION** | Compact D3/electric ray pass |
| Standard factorized SK | Contour legs are F1/D1 species | **OBSTRUCTED** | Independent tensor factors; diagonal normalization |
| Duality wall | Wall chooses its own modular label | **NO SELECTION** | Label defines the wall |
| Displayed $SO(8)$ stack | D7/O7 monodromy is inverse-$S$ | **FAIL** | Net monodromy $-I$ |
| Homogeneous same-$\tau$ wall alone | Stabilizer selects $I$ | **CONDITIONAL AT V1.0** | True only before real-structure compatibility |
| Primitive same-$\tau$ real-equivariant frame | $\pm X$ and $C_T$ are integrally conjugate | **OBSTRUCTED** | Every intertwiner determinant even |
| $dP_5$ index-two rescue | Minimal equivariant image is full visible frame | **FAIL** | Excludes primitive electric charge one |
| $\mathbb{Z}_2$ saturation | Preserves original ordered R/A claim | **FAIL** | Diagonalizes only in half-sum/half-difference basis |
| Linear Poincare attachment | Full torus yields two degree-two charge lines | **OBSTRUCTED** | Wrong degree, circular subcircle choice, or rank collapse |
| Local Hopf-cup BF wall | Cup product yields an inverse-$S$ parent locally | **CONDITIONAL PASS** | Restricted minimal wall class and assumed torus bundle |
| Published time-only sky equator | $g(k, U) = 0$ defines a loop | **FAIL / CORRECTION** | No nonzero null solution |
| Wall-normal sky loop | Timelike wall supplies an equator | **PASS** | Coorientation and tangent $U$ required |
| Natural spherical torus bundle | Primitive fiber area globalizes | **OBSTRUCTED** | Half-Euler class $\ell = 1$ |
| Ordinary normal/homogeneous duality line | Existing parent cancels $\ell$ | **FAIL** | Chern class zero |
| D3 monopole compensator | Unit flux is neutral and stressless | **FAIL MINIMALLY** | D1 charge and positive stress |
| $SU(2)_R$ Hopf twist | Opposite eigenlines cancel the Chern pair | **BUNDLE-LEVEL PASS** | Representation and physical action still open |

| CANDIDATE | TESTED PROPOSITION | RESULT | SCOPE |
|---|---|---|---|
| Independent R background | Matter stress is conserved by itself | **FAIL GENERICALLY** | Residual $F_{\mathcal{B}}J_{\mathcal{B}}$ force |

### Selector no-go theorem for the audited record

Within the explicitly tested class—integral pairing, balanced endpoint energies, the published CICY parent and its ambient-linear orientifold translation, static global $dP_5$ data, standard factorized SK doubling, homogeneous transparent walls, primitive real-structure equivariance, the stated linear differential-cohomology kernels, and the unmodified natural spherical Hopf-torus bundle—there is no unconditional derivation of the ordered D3 charge frame.

The inverse-$S$ Hopf-cup wall and $SU(2)_R$ compensator are not counterexamples to this theorem because their remaining physical premises are explicitly conditional. They form the surviving construction route.

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## 15. Considered, deferred, and unexplored routes

Nothing in this section is part of the no-go theorem unless a separate row states a proved obstruction.

| ROUTE | STATUS | WHAT WOULD BE REQUIRED |
|---|---|---|
| Nonambient or birational $\Omega$-odd involution of the #7447 quotient | **UNEXPLORED** | Explicit involution, $\Omega$ action, fixed loci, resolution, tadpoles, brane spectrum |
| O5-type use of the regular $F = +1$ quotient symmetry | **UNEXPLORED** | Consistent projection, tadpoles, surviving gauge fields, endpoint map |
| Nongeometric heterotic/Type-IIB dual of the monad model | **UNEXPLORED** | Charge/moduli/spectrum dictionary to a D3 F1/D1 lattice |
| Fourier–Mukai/Mukai selection of a D3 plane from the period lattice | **UNEXPLORED** | Integral physical functor selecting one primitive rank-two plane |
| Different compact geometry or free quotient | **UNEXPLORED** | $\Omega$-odd orientifold data and a visible D3 sector |
| Flux-stabilized $dP_5$ vacuum at the benchmark coupling | **UNEXPLORED** | Flux integers, moduli solution, tadpoles, kinetic matrix and RG matching |
| Exact global Wilson–'t Hooft lattice of the $dP_5$ model | **OPEN / NOT COMPUTED** | Center quotients, Stückelberg reduction, magnetic lattice and mixing |
| Closure attached only to a proper index-two dyonic sublattice | **UNEXPLORED DIFFERENT PARENT** | New endpoint action and treatment of primitive electric matter outside the image |

| ROUTE | STATUS | WHAT WOULD BE REQUIRED |
|---|---|---|
| Physical $\mathbb{Z}_2$ saturation of the closure lattice | **OPEN ALTERNATIVE** | Spin/quadratic-refinement or discrete-B-field origin and revised ordered channels |
| Nonfactorizing SK/charge defect | **UNEXPLORED** | Microscopic gauge-invariant influence functional, diagonal normalization, PSD noise and integral map |
| Non-topological interface with propagating modes | **UNEXPLORED** | Full wall action, stress, mode count, anomalies and noise |
| Different seven-brane word with genuine $S$ monodromy | **UNEXPLORED** | Global monodromy, tadpoles, D3 intersection and closure attachment |
| Degree-three gerbe kernel | **UNEXPLORED** | Integral pushforward, gauge invariance, rank-two retention and physical carrier |
| Higher-form or KK carrier in an enlarged 10D truncation | **UNEXPLORED** | Parent field inventory, orientifold projection, harmonic expansion and constraint audit |
| Relative differential class/anomaly inflow replacing absolute $u$ | **OPEN** | Quantized relative cocycle, inflow action, edge sector and anomaly cancellation |
| Framed/toroidal activation wall | **TOPOLOGICAL PASS; PHYSICAL OPEN** | Dynamics producing an $\ell = 0$ wall with the required source geometry |
| Open wall patches with edge modes | **UNEXPLORED** | Overlap action, edge constraints, Ward closure and PSD noise |
| Different covariant sky-loop selector | **UNEXPLORED** | Nonvanishing spacelike datum, zeros/turning points and rank audit |
| Monodromic half-duality compensator | **OPEN** | $c_1(L_{\mathcal{D}}) = -2$, chiral defects, half-charge soldering and inflow |
| Sourced D1–D3 monopole compensator | **OPEN NONMINIMAL** | D1 source, stress/tadpoles, line operators and renewed selector audit |
| Dynamical $SU(2)_R$ texture | **OPEN** | Full sigma-model/D3 action, Hessian, modes, stress and noise |
| Composite normal-bundle $SU(2)_R$ twist | **PREFERRED REMAINING ROUTE** | Closure representation, kappa/supersymmetry projector, embedding equations, improved Ward tensor, anomalies, inverse coupling |
| Full F-theory duality/R twist | **OPEN ALTERNATIVE** | 7-brane/defect geometry, duality line, R-line, chiral inflow and spherical Lorentzian realization |

A route that has not been built is not evidence for either viability or failure.

## 16. Self-correction ledger

| ID | EARLIER STATEMENT OR INFERENCE | CORRECTION | WHAT SURVIVES |
|---|---|---|---|
| C-1 | CICY #7447 was associated with $(3,75)$ | Exact cover is $(5,45)$; quotient is $(1,5)$, $\widehat{H}^3=12$ | Corrected quotient and heterotic bundle programme |
| C-2 | Exact quotient monodromies were treated as unavailable | Integral rank-four periods and monodromies exist | They still do not select a D3 plane |
| C-3 | Nonempty quotient-compatible polynomial spaces implied conditional O3/O7 feasibility | The residue character is $+1$; the cover parity does not descend | Regular quotient symmetry survives; O3/O7 branch closes in the audited class |
| C-4 | The first $(F_D, F)$ coordinate was called positive electric | It is D1/magnetic; F1/electric is the second coordinate | Integer minimum theorem survives; ordered interpretation changes |
| C-5 | Fixing the electric primitive selected $I$ | Advanced-electric gives $I$; retarded-electric gives $-J$ | Exact ordered fork |
| C-6 | The upper-shear $\alpha^{-1}$ gap was the global isolation gap | The lower shear gives gap $\alpha$; exact BPS gap is $3.11682355 \times 10^{-4}$ | Fourfold minimum survives |
| C-7 | More compactification data were expected to label R/A | The global $dP_5$ parent gives a primitive electric ray but no causal column | Compact D3 parent survives; static selector fails |
| C-8 | SK orientation might identify F1 and D1 | SK and charge are independent tensor factors; split charges violate $Z[A,A]=1$ | Standard SK–D3 parent remains consistent |
| C-9 | A wall might implement and select its label | A wall implements a chosen label; coupling continuity was only a conditional discriminator | Exact wall module and stabilizer theorem survive |
| C-10 | The $4\pi^2$ pairing could itself produce a closure photon and coupling | Fiber degree gives a scalar; topology does not fix $\tau_{\mathrm{cl}}$ | Normalized cup product survives as a wall level |
| C-11 | $\tau = i\alpha^{-1}$ was sometimes treated as compactification-derived | Neither compact parent derives the numerical low-energy coupling | It remains a declared benchmark |
| C-12 | Same-coupling transparency plus positive orientation could complete $K=I$ | Primitive closure exchange and ordinary D3 time reversal are not integrally conjugate | Stabilizer theorem remains true; the primitive transparent parent fails real equivariance |

| ID | EARLIER STATEMENT OR INFERENCE | CORRECTION | WHAT SURVIVES |
|---|---|---|---|
| C-13 | A rank-two linear differential-cohomology map was the preferred attachment | Degree bookkeeping blocks it or makes it circular | The bilinear area class supplies a primitive BF level instead |
| C-14 | Time orientation defined the sky equator by $g(k, U) = 0$ | No nonzero null vector is orthogonal to timelike $U$ | Hopf torus theorem survives for an independently supplied loop; wall normal repairs it locally. The published paper carries a dated erratum recording this correction |
| C-15 | An oriented torus bundle automatically supplied $\pi_* u = 1$ | The primitive area transgresses unless $\ell = e(P)/2 = 0$ | Local BF audit survives; naive spherical globalization fails |
| C-16 | A generic compensator line might repair the spherical wall | The primitive diagonal line is unique; ordinary normal, homogeneous duality line and neutral monopole do not supply it | Search narrows to a derived inverse spin/R line or inflow |
| C-17 | Bundle cancellation would complete the parent | The $SU(2)_R$ twist cancels topology but leaves representation, Ward, rank, anomaly and UV origin open | Global bundle-level pass is retained without status inflation |

## 17. Graded claim ledger

| ID | CLAIM | GRADE | BASIS |
|---|---|---|---|
| V2-1 | $\mathcal{M}(\tau)$ is positive with determinant one | **THEOREM** | Direct algebra |
| V2-2 | $\mathrm{Sym}^2 \sqrt{\mathcal{M}}$ has reciprocal unordered spectrum | **THEOREM** | Symmetric-square representation |
| V2-3 | The benchmark gives $\{\alpha^{-1}, 1, \alpha\}$ | **DERIVED, CONDITIONAL INPUT** | Assumed $\tau = i\alpha^{-1}$ |
| V2-4 | Integrality and symplectic pairing select the frame | **FALSE / COUNTEREXAMPLE** | Primitive shear |
| V2-5 | The stated endpoint energies have exactly | **THEOREM FOR STATED ACTIONS** | Integer proof and enumeration |

| ID | CLAIM | GRADE | BASIS |
|---|---|---|---|
| | four minima | | |
| V2-6 | The current parent derives those endpoint energies | **OPEN** | Physical closure-string identification absent |
| V2-7 | Advanced-electric gives $I$; retarded-electric gives $-J$ | **CONDITIONAL THEOREM** | Corrected fixed-column minimization |
| V2-8 | The two frames have the same ordered weights | **FALSE** | Outer entries reverse |
| V2-9 | The existing CICY flavour construction is a direct D3 model | **FALSE** | Heterotic monad bundle |
| V2-10 | The smooth quotient admits the standard genus-one duality shortcut | **OBSTRUCTED** | Rank-one cubic |
| V2-11 | The audited ambient quotient admits an O3/O7 branch | **OBSTRUCTED IN CLASS** | 26,624-case $\Omega$-character audit |
| V2-12 | The independent $dP_5$ model supplies a compact D3 sector and primitive electric ray | **DERIVED / SOURCE-SUPPORTED** | Tadpoles and spectrum |
| V2-13 | Static compactification data select R/A | **NO-SELECTION THEOREM FOR DATA CLASS** | No causal label |
| V2-14 | Standard SK and charge labels factorize | **THEOREM FOR STANDARD PARENT** | Tensor-product action |
| V2-15 | Opposite SK branches may carry F1 and D1 while preserving unitarity | **OBSTRUCTED** | Diagonal normalization residual |
| V2-16 | A specified Abelian wall implements $SL(2,\mathbb{Z})$ | **ESTABLISHED MODULE** | Quantized BF/CS walls |
| V2-17 | A wall chooses its own label | **FALSE** | Label defines the wall |
| V2-18 | Generic imaginary coupling has stabilizer $\{\pm I\}$ | **THEOREM** | Modular fixed-point equation |
| V2-19 | Primitive closure reversal and ordinary D3 time reversal are integrally conjugate | **FALSE / THEOREM** | All intertwiner determinants even |
| V2-20 | The minimal equivariant image contains primitive electric charge one | **FALSE / THEOREM** | Smallest pure electric charge is two |

| ID | CLAIM | GRADE | BASIS |
|---|---|---|---|
| V2-21 | The index-two pullback preserves the unordered ladder | **THEOREM** | Determinant-normalized spectrum |
| V2-22 | The same pullback preserves ordered $RR, RA, AA$ channels | **FALSE / THEOREM** | Nonzero R/A mixing |
| V2-23 | Preserving primitive ordered R/A requires an $S$ factor | **THEOREM WITH ENDPOINT MINIMA** | $C_T S = X$, $SC_T = -X$ |
| V2-24 | Full-torus linear line-kernel attachment yields two charge connections | **OBSTRUCTED IN KERNEL CLASS** | Degree and rank theorem |
| V2-25 | Normalized Hopf cup product has level one | **DERIVED** | $4\pi^2$ pairing |
| V2-26 | The restricted minimal CP-even wall is $\pm$ level-one BF | **CONDITIONAL THEOREM** | Two-field unimodular classification |
| V2-27 | Corrected orientation yields $K = -J$ | **DERIVED, CONDITIONAL DOMAIN** | R/A order plus wall direction |
| V2-28 | Fixed level-one BF has rank four and zero local modes | **THEOREM** | Complete wall constraint count |
| V2-29 | Its wall noise is zero and PSD | **THEOREM** | Pure topological phase |
| V2-30 | It recovers the ordered response and thresholds | **DERIVED** | Exact $-J$ pullback |
| V2-31 | Time orientation alone defines $g(k, U) = 0$ sky equator | **FALSE / CORRECTED** | Lorentzian null theorem |
| V2-32 | A cooriented timelike wall supplies $g(k, n) = 0$ equator | **THEOREM** | Spacelike wall normal |
| V2-33 | The natural torus bundle has Chern vector $(\ell, -\ell)$ | **THEOREM** | Spinor weights |
| V2-34 | Primitive fiber area globalizes iff $\ell = 0$ | **THEOREM IN NATURAL BUNDLE CLASS** | Leray–Serre transgression |
| V2-35 | Naive spherical wall passes this test | **FALSE / DERIVED** | $\ell = 1$ |
| V2-36 | A primitive diagonal compensator is unique | **THEOREM** | Integral Chern cancellation |
| V2-37 | Ordinary normal or homogeneous duality | **FALSE** | Chern class zero |

| ID | CLAIM | GRADE | BASIS |
|---|---|---|---|
| | line is that compensator | | |
| V2-38 | Unit D3 monopole is a neutral zero-stress compensator | **FALSE** | D1 endpoint and positive energy |
| V2-39 | Degree-one $SU(2)_R$ eigenlines have Chern numbers $\pm 1$ | **THEOREM** | Projector Chern formula |
| V2-40 | The diagonal $SU(2)_R$ twist cancels the spherical bundle obstruction | **THEOREM GIVEN ASSIGNMENT** | Exact Chern cancellation |
| V2-41 | Closure R/A phases carry the required opposite $SU(2)_R$ charges | **OPEN** | Soldering representation absent |
| V2-42 | A fixed R background closes the spatial Ward identity | **FALSE GENERICALLY** | $F_{\mathcal{B}} J_{\mathcal{B}}$ source |
| V2-43 | A natural composite twist has conserved improved stress classically | **CONDITIONAL THEOREM** | Diffeomorphism invariance of complete composed action |
| V2-44 | Dynamical/composite full Dirac rank and noise are safe | **OPEN** | Full action not constructed |
| V2-45 | Quantum R/duality/normal-bundle anomalies cancel | **OPEN** | Anomaly/inflow audit absent |
| V2-46 | The current D3 line is an unconditional global parent-level go | **NOT ESTABLISHED** | Representation, action, coupling and anomaly gates remain |

## 18. What is not established

This V2.0 paper does **not** establish:

1. that the closure cycles are physical $(p, q)$ strings ending on a D3;
2. that the Clifford Hodge norm is the D3 endpoint energy;
3. a compactification derivation of $\tau = i\alpha^{-1}$ or $i\alpha$;
4. a direct Type-IIB dual of the heterotic CICY #7447 flavour model;
5. a nonambient O3/O7 involution of the quotient;
6. a full Wilson–'t Hooft lattice for the $dP_5$ visible sector;
7. a nonfactorizing SK charge defect;

8. a physical $\mathbb{Z}_2$ saturation preserving the published claims;
9. a degree-three gerbe or higher-form alternative;
10. a global source producing the inverse-$S$ coupling pair;
11. a global R/A side assignment from the wall propagator;
12. constant rank at creation or destruction of the topological wall;
13. a relative-cocycle or edge-mode completion of the Euler obstruction;
14. a physical reason for a toroidal activation wall;
15. that closure R/A is an $SU(2)_R$ doublet;
16. a covariant soldering map from the closure Hopf spinors to D3 transverse eigenlines;
17. a Lorentzian kappa-symmetric D3 or five-brane configuration realizing the degree-one spherical twist;
18. the composite twist action and improved stress tensor;
19. constant full Hessian/Dirac rank when the twist, metric and embedding fluctuate;
20. positivity of the enlarged dynamical influence kernel;
21. cancellation of R-symmetry, electromagnetic-duality, gravitational, normal-bundle, brane and corner anomalies;
22. tadpole and line-operator completion of a monodromic or sourced alternative;
23. a completed global inverse-$S$ parent;
24. an unconditional microscopic ordered selector;
25. a framework-wide no-go or any status change elsewhere in STF.

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## 19. Exact current fork and next decisive construction

The old V1.0 fork was

$$\text{same coupling} \Rightarrow I, \qquad \text{inverse coupling} \Rightarrow -J.$$

After the real-structure theorem, the first branch is no longer viable for the primitive ordered R/A lattice. After the Hopf-cup construction, the second branch is no longer merely an abstract wall label. After the Euler and compensator audits, its current form is

$$\boxed{\begin{gathered}\text{primitive ordered closure lattice} \xrightarrow{\text{Hopf cup}} \text{level-one inverse-}S\text{ BF wall} \xrightarrow{\mathrm{SU}(2)_R\text{ twist}} \text{global spherical bundle,}\\ \text{but} \quad \text{closure representation} + \text{composite Ward/Dirac/anomaly/UV action} \quad \text{remain open.}\end{gathered}}$$

The remaining decisive route is therefore not another two-by-two selector search. It is one action-level construction of a D3 normal/R-symmetry or combined R/duality twist satisfying all of the following:

1. derive the opposite $SU(2)_R$ charges of the ordered closure pair;
2. derive the degree-one texture or normal-bundle connection from brane or defect geometry;
3. retain the level-one inverse-$S$ map and the coupling jump $i\alpha \leftrightarrow i\alpha^{-1}$;
4. include the D3 embedding, wall-frame and twist equations;
5. compute the complete primary Hessian and show constant constraint rank;
6. derive the improved Ward identity, including spatial components;

7. compute R/duality/normal-bundle anomaly inflow and corner terms;
8. derive the full SK kernel and prove noise positivity after all new modes are integrated or retained;
9. preserve the exact ordered three-channel response used by the activation and Floquet analyses;
10. provide global brane, tadpole and line-operator consistency.

Passing all ten would upgrade the inverse-$S$ line to a parent-level **GO**. Failure of an indispensable gate would close this spherical D3 realization, not the STF framework and not the unexplored routes in Section 15.

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## 20. Reproducibility

The single NumPy-only companion

`stf_d3_charge_frame_selector_record_v2_checks.py`

consolidates the decisive finite and algebraic checks:

- determinant-one D3 metric and reciprocal symmetric-square spectrum;
- primitive shear counterexample;
- fourfold BPS/flux minima and corrected nearest gaps;
- endpoint-orientation fork and ordered/unordered separation;
- corrected CICY invariants, fibration obstruction, period-plane ambiguity, and the 26,624-case equivariance enumeration;
- $dP_5$ tadpoles and primitive electric ray;
- SK factorization, diagonal residual and twisted-self-duality rank;
- duality-wall stabilizers and inverse coupling;
- integral real-structure nonconjugacy, index-two electric obstruction, channel mixing and $S$-twisted reversal;
- differential-degree obstruction, normalized Hopf level, BF uniqueness, Dirac rank, zero modes, large-gauge phases and zero noise;
- Lorentzian sky correction and wall-normal equator;
- spin-line Chern vector, Leray–Serre transgression, genus and torsion tests;
- unique compensator, homogeneous-duality and monopole tests;
- numerical $SU(2)_R$ projector Chern numbers, exact cancellation, texture energy, external-background Ward force, and unchanged ordered response.

All assertions pass in one run. Printed `FAIL-EXPECTED` lines are scientific grades, not program failures.

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## 21. Final assessment

The D3 line now contains three different kinds of result, and keeping them separate is the main conclusion of V2.0.

First, there is an exact invariant theorem: a determinant-one D3 duality metric produces a reciprocal unordered symmetric-square spectrum. At the benchmark it is $\{\alpha^{-1}, 1, \alpha\}$.

Second, there is a complete selector no-go record for the mechanisms that were actually tested. Integral pairing, balanced energy, the published CICY flavour parent, ambient O3/O7 descent, static $dP_5$ data, standard SK doubling, homogeneous transparent walls, and the primitive same-coupling real structure do not produce the ordered frame. The real-structure theorem is decisive: it removes the apparent transparent shortcut and makes an $S$ factor necessary if the primitive ordered R/A construction is kept.

Third, there is a surviving conditional construction. The normalized Hopf cup product supplies the primitive level of a local inverse-$S$ BF wall; the wall has the correct fixed-sector rank, Ward matching, noise and ordered response. The naive spherical globalization is obstructed by the unit half-Euler class, but a degree-one $SU(2)_R$ Hopf twist cancels that obstruction exactly and without mixing R/A.

That last result is a genuine bundle-level advance, not a full physical go. The closure pair has not been soldered to the $SU(2)_R$ eigenlines, and no complete D3/defect action yet supplies the twist, improved Ward tensor, constant dynamical rank, anomaly inflow, positive full kernel and dual coupling region.

The exact present verdict is

$$\boxed{\text{selector search closed for the audited direct routes;}\quad \text{inverse-}S + SU(2)_R \text{ route survives conditionally;}\quad \text{glob}}$$

The next calculation is therefore sharply defined: construct or rule out the composite spherical D3 R/duality-twist action. No additional relabelling of the unordered spectrum can substitute for that action.

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