Bell Autonomy and Superdeterminism in the Two-Clock STF Framework

Initial Setting-Effect Scalarization, Terminal Record Neutrality, and the Exact Status of STF V8.1

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Canonical synthesis of the thirteen-paper Bell-autonomy sequence

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Abstract

Does the Selective Transient Field framework prove or disprove superdeterminism? This paper consolidates the complete STF Bell-autonomy program developed after *First Principles V8.1* and gives the strongest answer currently licensed by the theory.

The answer remains neither, but the logical and mathematical content of that answer is now substantially stronger. STF V8.1 does not derive a correlation between the ontic Bell-source state and later analyzer settings; therefore its universal clock, global closure condition, and one-history ontology do not by themselves entail superdeterminism. Conversely, V8.1 does not yet derive the setting-generator sector or the terminal record dynamics needed to prove that every physical STF completion is non-superdeterministic.

The problem separates into two independent closures.

The **initial-autonomy closure** concerns the preterminal source and setting process. Let Λ_S be the gauge-invariant state of a physically defined source world tube on a universal-time leaf after source preparation and before the setting outputs. Let the complete setting process induce effects

$$K_{xy} \geq 0, \qquad \sum_{x,y} K_{xy} = I_S,$$

on the supported source algebra. Universal statistical source autonomy is equivalent to

$$\boxed{K_{xy} = q_{xy} I_S.}$$

In a diffeomorphism-constrained theory this scalarization has two independent parts: the effects must resolve neither the source microstate inside a shared boundary sector nor the sector label itself. Physical intervention autonomy additionally requires charge-preserving local replacement modules whose substitution leaves the source preparation process invariant. Exact universal decoupling is generated uniquely by a replacer channel. A gauge-compatible realization is a sector-preserving replacer with a neutral pointer doublet and sector-blind readout. Its microscopic realization requires a fresh or sectorwise-fresh resource; reset dynamics cannot select its own initially uncorrelated state, and no bounded finite-dimensional time-local generator becomes an exact replacer at finite time.

The **terminal-neutrality closure** concerns the future STF macrocondition. Let

$$E_R = \mathcal{J}^\dagger \left(U^\dagger P_{\mathrm{HD}} U \right)$$

be the heat-death effect pulled back through the complete physical preparation map to the regulated Bell record algebra. Full preservation of the prepared source and outcome distribution for a setting pair (x, y) is equivalent to

$$\boxed{\Pi_{xy} E_R \Pi_{xy} = c_{xy} \Pi_{xy}}$$

on the supported source-outcome block. Source preservation, no-signaling, setting-frequency preservation, and Born preservation are distinct conditions. A constructive diagonal terminal effect can preserve the source prior, uniform setting frequencies, and both no-signaling marginals while postselecting ordinary quantum records into a Popescu–Rohrlich box with CHSH value 4. Outcome-record scalarization is therefore indispensable.

The combined theorem is exact. If the initial setting effects are scalar, the natural and forced setting modules are genuine source-invariant replacements, and the terminal effect is scalar in every supported setting block, then the completed STF experiment preserves both the initial source prior and the supplied quantum Bell distribution. In that operational Bell sense the completion is non-superdeterministic.

FP V8.1 does not yet prove the premises. It specifies the clock-relative curvature norm

$$q_N = \sqrt{R^2 + 8\mathcal{W}_N}$$

and the exact selective response

$$K^R_{\mathrm{sel}}(t) = \delta(t) - \omega_c e^{-\omega_c t}\Theta(t), \qquad K_{\mathrm{sel}}(\omega) = -\frac{i\omega}{\omega_c - i\omega},$$

but it does not yet derive the setting resources, visible production operator, state-dependent noise kernel, irreversible record-transfer kernel, complete non-Markovian process tensor, fixed/peripheral algebra, or physical pulled-back terminal effect. Directly identifying the selective kernel with record population relaxation fails: every decaying mode retains the finite fraction

$$\frac{\omega_c}{\omega_c + \Gamma\lambda} > 0.$$

The exact status is therefore

$$\boxed{\mathrm{STF}_{\mathrm{V8.1}} \nvdash \mathrm{SD}, \qquad \mathrm{STF}_{\mathrm{V8.1}} \nvdash \neg\mathrm{SD}.}$$

What is now complete is the decision architecture: superdeterminism is no longer a semantic question inside STF. It has been reduced to calculable source-to-setting effects and terminal record effects derived from the physical theory.

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1. Question, scope, and result

The term *superdeterminism* is frequently allowed to absorb several different claims. Determinism is not by itself superdeterminism. A block-universe description is not by itself superdeterminism. A two-boundary theory is not by itself superdeterminism. Nor does the existence of one actual history establish the Bell-relevant dependence merely because that history contains the settings that occurred.

The Bell question is narrower. Did the physical variables that determine the prepared source become coordinated with the variables that determine the later local settings in a way that invalidates measurement independence or the operational meaning of changing those settings?

For STF, that question has two boundaries.

1. At the **initial boundary**, the source and setting mechanisms may share a common past, conserved sectors, or correlated resources.
2. At the **terminal boundary**, the heat-death macrocondition may reweight completed source-setting-outcome records.

A proof of non-superdeterminism requires both boundaries to be neutral in the relevant sense. Closing only one leaves the other available as a source of measurement dependence.

This paper does four things.

1. It defines the Bell-relevant STF source variable without importing the settings by definition.
2. It consolidates the exact initial-autonomy theorem chain into one operator criterion and one physical modularity criterion.
3. It consolidates the terminal-neutrality theorem chain into one pulled-back effect criterion, together with its dynamical obstructions.
4. It proves the combined two-boundary Bell-autonomy theorem and states the exact FP V8.1 proof boundary.

The result is a theorem-driven synthesis. The companion papers retain the full proofs, numerical audits, and regulator-specific calculations; every premise needed for the main conclusion is stated here.

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2. Bell measurement independence and three distinct propositions

Let X, Y be the setting labels, A, B the outcomes, and Λ_S the ontic source state. A Bell-local hidden-variable model has

$$P(A, B \mid X, Y, \Lambda_S) = P(A \mid X, \Lambda_S)P(B \mid Y, \Lambda_S).$$

The usual statistical measurement-independence condition is

$$\boxed{P(\Lambda_S \mid X, Y) = P(\Lambda_S),}$$

or equivalently

$$I(\Lambda_S : X, Y) = 0.$$

This is the assumption whose relaxation permits measurement-dependent local models of Bell correlations [18–21].

For an operational theory, this is not the whole requirement. The symbols $do(x)$ and $do(y)$ must refer to genuine physical replacements of local setting mechanisms. Intervention autonomy then requires

$$\boxed{P_0(\Lambda_S \mid do(x), do(y)) = P_0(\Lambda_S).}$$

These equations answer different questions. Statistical independence concerns the naturally sampled ensemble. Intervention autonomy concerns what changes when a local device module is physically replaced.

Three propositions must therefore be kept separate.

2.1 Initial/common-cause superdeterminism

The initial proposition is that the source and natural setting outputs are correlated before terminal reweighting:

$$I_0(\Lambda_S : X, Y) > 0,$$

or that the physical setting substitutions are not modular and change the source process.

2.2 Terminally induced measurement dependence

Even if the preterminal experiment is autonomous, a future macroeffect may assign unequal weights to different source labels or outcome records. The postselected distribution can then satisfy

$$P_f(\Lambda_S \mid do(x), do(y)) \neq P_0(\Lambda_S).$$

This is the contribution unique to the two-boundary architecture.

2.3 Complete-history dependence

Let Γ_* denote the complete actual history. Since Γ_* contains the actual settings,

$$I(\Gamma_* : X, Y) > 0$$

is true by construction. It would also be true in an ordinary independently randomized Bell experiment if its entire realized history were used as the hidden variable. This is not a discriminating test of superdeterminism.

The Bell variable must instead represent the physical source information available in the source region, not the complete future history.

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3. The STF source variable and the two clocks

Let T_U denote the universal ordering variable and let local internal clocks order laboratory operations and records. The two-clock architecture allows the source, setting devices, detections, and terminal macrocondition to be assigned to distinct physical regions without identifying the local source state with the completed universal history [2,3].

Let $\mathcal{W}_S$ be the source world tube. Choose a universal-time leaf Σ_S that intersects $\mathcal{W}_S$ after source preparation and before the setting outputs. Define

$$\boxed{\Lambda_S = [g, \pi_g; \phi, \pi_\phi; \Psi, \pi_\Psi; \xi_{\text{prep}}]_{\Sigma_S \cap \mathcal{W}_S} / \text{gauge}.}$$

The definition includes the physical gravitational, STF-scalar, matter, and preparation data that determine the source state on that leaf. It excludes:

- the later setting labels X, Y;
- the later setting-resource microstates unless they already lie in the source world tube;
- the detector outcomes A, B;
- the completed terminal record; and
- the complete actual history Γ_*.

Shared boundary charges are not silently discarded. They appear as central sectors of the gauge-invariant source algebra and must be tested explicitly.

The clocks play distinct roles. The universal clock orders the source leaf, setting events, record formation, and terminal leaf. Internal clocks define the operational timing and phase of local devices. Neither clock by itself generates setting independence, scrambling, or a probability measure. Clock separation prevents a category error; it does not close the autonomy theorem.

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4. Why statistical and intervention autonomy are both required

The intervention equation alone does not exclude a common cause. Let G be a fair bit, set $\Lambda_S = G$, and let the settings tend to copy G:

$$P(X = G \mid G) = 0.9, \qquad P(Y = G \mid G) = 0.85.$$

The causal structure is

$$G \longrightarrow \Lambda_S, \qquad G \longrightarrow X, \qquad G \longrightarrow Y.$$

Replacing the setting equations cuts the arrows into X, Y and leaves the source unchanged, so

$$P_0(\Lambda_S \mid do(x), do(y)) = P_0(\Lambda_S).$$

Nevertheless,

$$I(\Lambda_S : X, Y) = 0.6813316047 \text{ bits}.$$

Conversely, observational independence can conceal intervention dependence. Let H^- be fair, take $X = H^-$ naturally, and define $\Lambda_S = X \oplus H^- = 0$. Then the natural ensemble has $I(\Lambda_S : X, Y) = 0$. Under $do(X = x)$, however, $\Lambda_S = x \oplus H^-$ becomes fair, giving a source shift of

$$\mathrm{TV}[P_0(\Lambda_S \mid do(x)), P_0(\Lambda_S)] = \frac{1}{2}.$$

Therefore

$$\boxed{\text{statistical autonomy} + \text{physical intervention autonomy}}$$

is a conjunction. Neither term implies the other.

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5. Cosmic Bell tests and the STF question

Ordinary Bell tests reject local hidden-variable explanations only under a set of auxiliary premises that includes measurement independence. Cosmic Bell tests use photons from distant astronomical sources to choose settings [25–27]. They push any ordinary common cause coordinating the source with the setting generators progressively farther into the shared past and make familiar local coordination mechanisms increasingly implausible.

They do not prove the logical impossibility of a theory in which the complete initial condition or a global two-boundary constraint coordinates all records. Nor do they derive a physical do-operation for an arbitrary ontology.

In STF, distance is not irrelevant. The distant emitter, propagation region, local receiver, and setting device occupy different world tubes, and their common causal past changes with the chosen astronomical source. The final setting is indeed materialized as a local record at the detector, but that does not erase the causal significance of the signal's prior propagation or make every distance physically equivalent.

The STF interpretation is therefore precise:

- Cosmic Bell results constrain ordinary source-setting common-cause models and any STF completion that reduces to them.
- They do not calculate the STF setting effects K_{xy}.
- They do not calculate the terminal effect E_R.
- They cannot replace either the initial-autonomy or terminal-neutrality theorem.

Part I — Initial Source–Setting Autonomy

Sections 6–17 synthesize the initial-autonomy companion sequence [5–8].

6. Common-past screening and the History-Balance Theorem

Let H^- contain the physically relevant information in the common past of the source and the two setting generators. Define

$$Q_{xy}(h) = P_0(X = x, Y = y \mid H^- = h).$$

Assume common-past screening:

$$P_0(x, y \mid \lambda, h) = Q_{xy}(h).$$

This assumption says that, once the relevant common-past data are fixed, the source label supplies no additional information about the setting output.

Theorem 1 — Exact History-Balance Criterion

Under common-past screening,

$$I_0(\Lambda_S : X, Y) = 0$$

if and only if

$$\boxed{\sum_h P_0(h \mid \lambda) Q_{xy}(h) = P_0(x, y) \quad \forall \lambda, x, y.}$$

Proof

Screening gives

$$P_0(x, y \mid \lambda) = \sum_h Q_{xy}(h) P_0(h \mid \lambda).$$

Statistical independence is exactly the requirement that this conditional probability equal $P_0(x, y)$ for every supported λ, x, y. □

Equivalently,

$$\sum_h [P_0(h \mid \lambda) - P_0(h)] Q_{xy}(h) = 0.$$

The setting kernel need not be pointwise history-independent. It must be orthogonal to every source-resolving change in the common-past distribution. A history-blind channel,

$$Q_{xy}(h) = P_0(x, y),$$

is a stronger sufficient condition that works for every possible source/common-past correlation.

7. Fresh stochasticity is not history balance

Let H^- be a fair bit, $\Lambda_S = H^-$, and let a fresh innovation Ξ_X satisfy

$$\Xi_X \perp (H^-, \Lambda_S), \qquad P(\Xi_X = 0) = p.$$

Set

$$X = H^- \oplus \Xi_X$$

and let Y be independent and fair.

Theorem 2 — Fresh-Stochasticity No-Go

For this construction,

$$\boxed{I(\Lambda_S : X, Y) = 1 - h_2(p),}$$

where h_2 is the binary entropy. The history-balance error is

$$\varepsilon_{\text{bal}} = \left| p - \frac{1}{2} \right|.$$

At $p = 0.8$,

$$I(\Lambda_S : X, Y) = 0.2780719051 \text{ bits}, \qquad \varepsilon_{\text{bal}} = 0.3.$$

The innovation is fresh, nondegenerate, and generated after the source, yet its output retains source information. At $p = 1/2$, the XOR output is fair and history-blind. The relevant word is therefore *balanced*, not merely *stochastic*.

The STF theorem of the unknown, or the existence of an irreducible bit of unpredictability, cannot by itself establish measurement independence. One bit of uncertainty is not the same statement as zero source-to-setting channel capacity.

8. Physical modularity and the process-tensor criterion

A setting is physically intervenable only if the local device admits a module replacement. In a structural description, changing

$$F_X \mapsto x, \qquad F_Y \mapsto y$$

must leave the common-past distribution, source exogenous state, source preparation map, and universal boundary rule unchanged.

Theorem 3 — Modular Source Invariance

If the replacements act only on the setting mechanisms, then

$$\boxed{P_0(\Lambda_S \mid do(x), do(y)) = P_0(\Lambda_S).}$$

The source marginal contains no replaced setting map, so the result follows immediately. This theorem supplies intervention autonomy, not statistical autonomy.

In process-tensor language [24,29,30], let $\Upsilon_{S;XY}$ be the preterminal process connecting the source region to the two setting slots. For admissible deterministic maps $\mathcal{M}_X, \mathcal{M}_Y$, define the induced source state $\rho_S[\mathcal{M}_X, \mathcal{M}_Y]$. Source non-signaling from the setting slots is

$$\boxed{\rho_S[\mathcal{M}_X, \mathcal{M}_Y] = \rho_S}$$

for every admissible replacement pair. A strictly forward-causal preterminal process has this property automatically. A globally conditioned two-boundary process must be rechecked after the terminal effect is included.

9. The conjunctive Source–Setting Autonomy Theorem

Theorem 4 — Initial Bell Autonomy

Suppose:

1. Λ_S is the source-world-tube state defined in Section 3;

2. common-past screening holds;
3. the setting kernel satisfies exact history balance;
4. natural and forced settings are genuine local module replacements;
5. the source process is invariant under those replacements; and
6. every setting pair used in the Bell functional has nonzero support.

Then

$$\boxed{I_0(\Lambda_S : X, Y) = 0}$$

and

$$\boxed{P_0(\Lambda_S \mid do(x), do(y)) = P_0(\Lambda_S)}$$

for every supported intervention.

Proof

Common-past screening and history balance give statistical independence by Theorem 1. Physical module replacement and process invariance give intervention autonomy by Theorem 3. Full support ensures the experiment actually samples all terms used in its Bell functional. □

Wing independence remains separate. Source autonomy does not imply $P(x, y) = P(x)P(y)$. A fresh source-independent bit copied to both wings gives $I(\Lambda_S : X, Y) = 0$ but $I(X : Y) = 1$ bit. Independent Bell settings require the additional factorization and support conditions

$$q_{xy} = q_X(x)q_Y(y) > 0.$$

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10. Universal decoupling and the unique replacer channel

The history-balance theorem characterizes the required output statistics but does not identify a local channel that guarantees them for arbitrary incoming correlations. Let R be the incoming setting register, O the output register, and

$$\Phi : \mathcal{B}(\mathcal{H}_R) \to \mathcal{B}(\mathcal{H}_O)$$

be a completely positive, trace-preserving map.

The argument uses only standard finite-dimensional channel duality and complete positivity [34].

Theorem 5 — Replacer-Channel Equivalence

For a fixed output state τ_O, the following are equivalent.

1. For every finite reference S and every joint state ρ_{SR},

$$(\mathrm{id}_S \otimes \Phi)(\rho_{SR}) = \rho_S \otimes \tau_O.$$

2. Every input density operator is mapped to τ_O.
3. For every operator A,

$$\boxed{\Phi(A) = \tau_O \operatorname{Tr} A.}$$

Proof

The third statement immediately gives the first by a product-operator expansion of ρ_{SR}. The first gives the second by taking a trivial reference. If every density operator is sent to the same state, linearity on their affine span yields the third statement. $\square$

The map

$$\mathcal{R}_\tau(A) = \tau \operatorname{Tr} A$$

is the replacer channel. It has zero capacity to transmit any property of the incoming register into the exposed setting output. High entropy, chaos, and unpredictability do not imply this zero-capacity property.

For two local replacers,

$$(\mathrm{id}_S \otimes \mathcal{R}_X \otimes \mathcal{R}_Y)(\rho_{SR_XR_Y}) = \rho_S \otimes \tau_X \otimes \tau_Y.$$

Choosing $\tau_X = \tau_Y = I_2/2$ and reading in the computational basis gives

$$P_0(x, y) = \frac{1}{4}, \qquad I(S : X, Y) = 0, \qquad I(X : Y) = 0.$$

The natural setting module is $\mathcal{R}_{I_2/2}$. A forced intervention is the deterministic replacer

$$\mathcal{D}_{j,a}(A) = |a\rangle\langle a| \operatorname{Tr} A.$$

Natural and forced settings therefore belong to one replaceable local module family.

The replacer is the unique **universal** architecture. A nonconstant channel can still be balanced on one actual ensemble, but it fails for some admissible source/input correlation. In the classical case, universal decoupling is equivalent to identical transition-matrix rows:

$$W(o \mid r) = q(o) \quad \forall r, o.$$

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11. SWAP realization, fresh resources, and the reset regress

An exact replacer can be implemented by a SWAP with a fresh ancilla. Let E be prepared in τ and initially satisfy

$$\rho_{SRE} = \rho_{SR} \otimes \tau_E.$$

After swapping R and E, the source correlation formerly carried by R moves into E, while the exposed register carries τ. Tracing the old register gives

$$\rho_{SR}^{\text{out}} = \rho_S \otimes \tau_R.$$

The information has not been annihilated; it has been exported to the environment.

If the ancilla is source-correlated, the same SWAP imports that correlation into the setting output. For two wings define

$$\delta_{\text{anc}} = \frac{1}{2} \|\rho_{SE_XE_Y} - \rho_S \otimes \tau_X \otimes \tau_Y\|_1.$$

Theorem 6 — Exact SWAP Freshness Identity

For ideal local SWAP-and-discard implementations,

$$\boxed{\delta_{\text{out}} = \delta_{\text{anc}}.}$$

SWAP reproduces exactly the freshness of the resource it is given. It cannot create an initially factorized resource from a correlated one.

With implementation errors

$$\eta_j = \frac{1}{2} \|\widetilde{\mathcal{S}}_j - \mathcal{S}_j\|_\diamond,$$

the two-wing output obeys

$$\boxed{\delta_{\text{out}} \le \delta_{\text{anc}} + \eta_X + \eta_Y.}$$

Resetting a correlated register by swapping it with a fresh one merely moves the correlation to the new discarded register. Repeating the procedure creates a resource chain. An irreversible reservoir can disperse the record into inaccessible degrees of freedom, but its state, renewal law, free-energy cost, and boundary conditions must be supplied by the physical theory.

A reset interaction cannot select its own independent initial environment state. In a Stinespring representation,

$$\Phi(\rho_R) = \mathrm{Tr}_B \left[U(\rho_R \otimes \sigma_B)U^\dagger\right],$$

the state σ_B and its relation to the source are premises of the reduced channel, not consequences of the unitary U.

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12. Exact finite-time reset is not generated by bounded time-local mixing

Let a finite-dimensional register obey

$$\dot{\rho}(t) = \mathcal{L}_t[\rho(t)]$$

with a bounded time-local generator on $[0, t_*]$. Its propagator

$$\Phi_{t_*,0} = \mathcal{T} \exp\left(\int_0^{t_*} \mathcal{L}_t dt\right)$$

is invertible as a linear map on operator space. A replacer has a nontrivial kernel because it annihilates every traceless input operator.

Theorem 7 — Bounded Finite-Time Reset No-Go

$$\boxed{\text{No bounded finite-dimensional time-local generator becomes an} \text{exact} \text{replacer at finite time.}}$$

Exact finite-time replacement therefore requires a singular generator, an explicit unitary-and-discard realization, a measurement-and-repreparation instrument, a non-Markovian finite-environment process, or an idealized discontinuous control operation.

For the depolarizing comparison channel

$$\Phi_t = e^{-\gamma t}\,\mathrm{id} + (1 - e^{-\gamma t})\mathcal{R}_{I/2},$$

the residual remains nonzero at every finite time. For a qubit,

$$\frac{1}{2}\|\Phi_t - \mathcal{R}_{I/2}\|_\diamond = \frac{3}{4}e^{-\gamma t}.$$

The Choi rank of a replacer from a d_R-dimensional input to target τ is

$$\boxed{r_{\mathrm{Choi}} = d_R\,\mathrm{rank}\,\tau.}$$

A uniform qubit replacer therefore requires pure Stinespring environment dimension at least four; a forced pure-setting replacer requires at least two.

These results do not make autonomy impossible. They identify the resources that an exact physical autonomy claim must expose.

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13. Gauge constraints and sectorwise freshness

Literal global tensor-product factorization is too strong as a fundamental criterion in a gauge or gravitational theory. Regional physical algebras may share central charges or edge data [31].

Let the regulated source algebra decompose as

$$\mathfrak{A}_S \simeq \bigoplus_z \mathcal{B}(\mathcal{H}_{S,z}), \qquad \mathfrak{Z}_S = \bigoplus_z \mathbb{C}P_z.$$

The label z denotes a boundary or conserved sector. Exact sectorwise freshness requires product structure only inside each supported sector:

$$\rho^{(z)}_{SE_XE_Y} = \rho^{(z)}_S \otimes \tau^{(z)}_X \otimes \tau^{(z)}_Y.$$

The setting readout must also be sector-blind:

$$\mathrm{Tr}[M_x\tau^{(z)}_X] = q_X(x), \qquad \mathrm{Tr}[N_y\tau^{(z)}_Y] = q_Y(y)$$

for every supported z.

Theorem 8 — Sectorwise Autonomy

Sectorwise freshness plus sector-blind local readout gives

$$P_0(x, y \mid \lambda) = q_X(x) q_Y(y)$$

even when the sector label z is contained in Λ_S. Consequently,

$$I_0(\Lambda_S : X, Y) = 0, \qquad I_0(X : Y) = 0.$$

The source and setting resources may share a complete conserved-sector bit without the exposed setting labels carrying any source information. Shared sectors are harmless only when the readout is sector-blind or satisfies the weaker ensemble-specific history-balance identity.

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14. Pulled-back setting effects and initial scalarization

Compress the complete setting-resource, reset, pointer, readout, and environment process into a quantum-to-classical channel on the supported source algebra:

$$\mathcal{C}_{\text{set}}(\rho_S) = \sum_{x,y} \mathrm{Tr}(\rho_S K_{xy}) |xy\rangle\langle xy|,$$

where

$$K_{xy} \geq 0, \qquad \sum_{x,y} K_{xy} = I_S.$$

The effect K_{xy} is the proposition "the setting pair will be (x, y)" pulled backward through the complete setting process to the source leaf.

The channel exists on the supported source algebra only if operationally equivalent source preparations give the same setting probabilities. If initial correlations prevent this, one must restrict to the actual compatibility domain or enlarge the upstream algebra to include the common-past degrees that physically reach the device [32,33]. Failure to define the map is itself an initial-correlation obstruction.

Theorem 9 — Initial Setting-Effect Scalarization

For every supported source state, the setting distribution is the same fixed distribution q_{xy} if and only if

$$\boxed{P_{\text{supp}} K_{xy} P_{\text{supp}} = q_{xy} P_{\text{supp}} \quad \forall x, y.}$$

Proof

Scalar effects plainly have state-independent expectation values. Conversely, if the expectation of K_{xy} is identical in every supported pure state, the polarization identity makes every off-diagonal matrix element vanish and every diagonal element equal the same number. Thus the effect is scalar on the supported space. □

For one realized source ensemble $\{p_\lambda, \rho_\lambda\}$, the weaker exact condition is

$$\mathrm{Tr}(\rho_\lambda K_{xy}) = \sum_{\lambda'} p_{\lambda'} \, \mathrm{Tr}(\rho_{\lambda'} K_{xy}).$$

This is the operator form of history balance. The scalar theorem is the universal, non-fine-tuned form.

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15. Boundary-center decomposition

Let $\mathbb{E}_3$ be the conditional expectation from the source algebra onto its center. Then

$$\boxed{K_{xy} - q_{xy} I = [K_{xy} - \mathbb{E}_3(K_{xy})] + [\mathbb{E}_3(K_{xy}) - q_{xy} I].}$$

The first term is within-sector microstate leakage. The second is center or sector leakage.

Define

$$q_{xy}(z) = \frac{1}{d_z} \mathrm{Tr}_{S,z}(P_z K_{xy} P_z).$$

Theorem 10 — Two-Stage Initial Scalarization

The global setting effect satisfies $K_{xy} = q_{xy} I$ if and only if both

$$\boxed{P_z K_{xy} P_z = q_{xy}(z) P_z}$$

for every supported sector and

$$\boxed{q_{xy}(z) = q_{xy}}$$

for every supported sector.

The first condition erases microstate information within each sector. The second prevents the exposed setting label from reading the shared boundary sector.

With

$$\delta_{\mathrm{micro}} = \max_z \sup_{\rho_z} \frac{1}{2} \sum_{x,y} |\mathrm{Tr}(\rho_z K_{xy}) - q_{xy}(z)|$$

and

$$\delta_{\mathrm{center}} = \max_z \frac{1}{2} \sum_{x,y} |q_{xy}(z) - q_{xy}|,$$

every supported source state obeys

$$\frac{1}{2} \sum_{x,y} |\mathrm{Tr}(\rho_S K_{xy}) - q_{xy}| \leq \delta_{\mathrm{micro}} + \delta_{\mathrm{center}}.$$

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16. Charge-preserving natural and forced setting modules

A global replacer may change a conserved boundary charge. The gauge-compatible replacement is sector preserving:

$$\boxed{\mathcal{R}_{\mathrm{sec}}(\rho) = \bigoplus_z \mathrm{Tr}(P_z \rho) \tau^{(z)}.}$$

It preserves every sector probability while deleting all within-sector input microstate information.

Introduce a neutral binary pointer Q_j in each sector and choose

$$\tau_j^{(z)} = \sigma_{z,j} \otimes \frac{I_{Q_j}}{2}.$$

The natural module is

$$\mathcal{N}_j(\rho) = \bigoplus_z \mathrm{Tr}(P_z \rho) \left(\sigma_{z,j} \otimes \frac{I_{Q_j}}{2} \right),$$

and the forced module is

$$\mathcal{D}_{j,a}(\rho) = \bigoplus_z \mathrm{Tr}(P_z\rho)\,(\sigma_{z,j} \otimes |a\rangle\langle a|).$$

Both preserve z, erase the incoming microstate, and differ only in the neutral pointer preparation. They therefore supply a concrete, charge-preserving meaning to $do(j = a)$.

This construction proves that a non-superdeterministic initial architecture is mathematically compatible with gauge constraints. It does not prove that FP V8.1 dynamically selects this architecture.

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17. Exact status of the initial branch

The initial branch is mathematically closed by

$$\boxed{K_{xy} = q_{xy} I_S}$$

on the supported source algebra, together with physical source invariance under the natural-to-forced module replacements.

This one statement collects four levels of description:

$$\text{replacer} \implies \text{history blindness} \implies \text{history balance} \implies I_0(\Lambda_S : X, Y) = 0.$$

The charge-preserving module then supplies the intervention meaning.

FP V8.1 does not yet derive:

- the physical setting registers;
- their boundary sectors and operational source algebra;
- the microscopic setting resources and their state;
- the reset or SWAP dilation and renewal mechanism;
- the natural and forced pointer maps;
- the effects K_{xy}; or
- source-process invariance under those replacements.

Therefore

$$\boxed{\text{initial-autonomy criterion: proved,}\qquad \text{physical STF realization: open.}}$$

Part II — Terminal Bell-Record Neutrality

Sections 18–31 synthesize the terminal-neutrality and late-time-dynamics sequence [9–16], with the FP V8.1 notation fixed by the alignment note [17].

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18. Two-boundary probabilities and complete branch weights

Let C_r denote mutually decoherent record histories on a record-forming leaf and define

$$X_r = C_r \rho_I C_r^\dagger, \qquad q_r = \operatorname{Tr} X_r, \qquad \sigma_r = \frac{X_r}{q_r}$$

for each nonzero branch. Let the terminal macrocondition be represented by a positive effect E_f. The two-boundary probability is

$$p_f(r) = \frac{\operatorname{Tr}(E_f X_r)}{\sum_s \operatorname{Tr}(E_f X_s)} = \frac{q_r w_r}{\sum_s q_s w_s},$$

where

$$w_r = \operatorname{Tr}(E_f \sigma_r).$$

Theorem 11 — Complete Branch Neutrality

For every branch with $q_r > 0$,

$$p_f(r) = q_r$$

if and only if

$$\boxed{w_r = w \quad \text{for every supported branch } r.}$$

Proof

Equal weights cancel from the common normalization. Conversely, if $p_f(r) = q_r$, division by q_r gives $w_r = \sum_s q_s w_s$ for every supported branch. □

This criterion concerns the complete physical branch, not merely an idealized record tensor factor. If

$$E_f = I_R \otimes E_E$$

but the environment state depends on the record,

$$\sigma_r = |r\rangle\langle r| \otimes \sigma_E^{(r)},$$

then

$$w_r = \mathrm{Tr}(E_E \sigma_E^{(r)})$$

can remain branch dependent. Commutation with the record algebra is therefore insufficient when branch information has migrated into the rest of the universe.

The finite audit gives an explicit example. With

$$q = (0.37, 0.63), \qquad w = (1, 0.05),$$

a perfectly record-commuting effect produces

$$p_f = (0.921544, 0.078456)$$

and total-variation bias 0.551544.

---

19. The complete preparation map and the effective terminal effect

Let τ be a regulated record-forming leaf ordered by the universal clock and T_f a regulated terminal leaf. Let

$$U(T_f, \tau)$$

be the complete propagator generated by the completed matter-gravity-clock- response theory, and let

$$0 \leq P_{\mathrm{HD}} \leq I$$

represent the terminal heat-death macrocondition.

Pull the terminal effect back to the record leaf:

$$F(\tau) = U^\dagger(T_f, \tau) P_{\mathrm{HD}} U(T_f, \tau).$$

Let $\mathcal{J}$ be the complete physical preparation channel from a finite regulated source-setting-record system R into the global state on that leaf. It includes detectors, environmental records, ordinary matter, gravitational dressing, and every branch correlation. Define

$$\boxed{E_R = \mathcal{J}^\dagger(F).}$$

Theorem 12 — Complete Pullback

For every regulated record input ρ_R, the terminal weight is exactly

$$\boxed{w(\rho_R) = \mathrm{Tr}(E_R \rho_R).}$$

Proof

By duality of the preparation channel,

$$\mathrm{Tr}[F\mathcal{J}(\rho_R)] = \mathrm{Tr}[\mathcal{J}^\dagger(F)\rho_R].$$

The right-hand side is the stated expression. □

This construction automatically retains branch-dependent environments. It does not assume a record/rest tensor product, factorized late-time state, or Haar-random propagator.

---

20. Exact terminal scalarization and its sharp error bound

Theorem 13 — All-State Terminal Scalarization

The terminal weight is independent of every supported record state if and only if

$$\boxed{P_{\mathrm{supp}} E_R P_{\mathrm{supp}} = f P_{\mathrm{supp}}.}$$

For one finite branch family $\{\rho_r\}$, the weaker necessary-and- sufficient condition is

$$\mathrm{Tr}[E_R(\rho_r - \rho_s)] = 0$$

for every supported pair r, s.

The all-state statement follows from the same polarization argument used for the initial setting effects. Thus the initial and terminal problems have the same algebraic form, but they act on different processes and answer different questions:

$$\boxed{K_{xy} = q_{xy} I_S \quad \text{initially,} \qquad \Pi_{xy} E_R \Pi_{xy} = c_{xy} \Pi_{xy} \quad \text{terminally.}}$$

Theorem 14 — Sharp Probability-Distortion Bound

Let

$$a = \lambda_{\min}(E_R) > 0, \qquad b = \lambda_{\max}(E_R).$$

For every prior q_r, branch family, and terminally reweighted distribution,

$$\boxed{\|p - q\|_{\mathrm{TV}} \le \frac{\sqrt{b} - \sqrt{a}}{\sqrt{b} + \sqrt{a}}.}$$

The bound is optimal. If

$$E_R = fI + \Delta, \qquad \|\Delta\|_\infty \le \delta < f,$$

then

$$\|p - q\|_{\mathrm{TV}} \le \frac{\delta}{f + \sqrt{f^2 - \delta^2}}.$$

To guarantee $\|p - q\|_{\mathrm{TV}} \le \epsilon$, it is sufficient that

$$\boxed{\frac{\delta}{f} \le \frac{2\epsilon}{1 + \epsilon^2}.}$$

This converts an experimental tolerance directly into the spectral flatness required of the physical pulled-back terminal effect.

---

21. Conserved sectors and the terminal center

If the late-time dynamics preserves exact sectors Q, the supported record algebra decomposes as

$$\mathcal{H}_R = \bigoplus_Q \mathcal{H}_Q.$$

Sectorwise neutrality requires

$$\boxed{P_Q E_R P_Q = f_Q P_Q.}$$

Different values f_Q are harmless only when every compared branch has the same sector profile. If branch r has sector weights p_{rQ}, its mean terminal weight is

$$w_r = \sum_Q p_{rQ} f_Q.$$

Unequal p_{rQ} combined with unequal f_Q creates a bias that does not vanish merely because each sector is large. The finite-record audit exhibits a persistent total-variation bias near 0.25 for a $3/4$ versus $1/4$ sector mismatch at dimension 1024.

The correct terminal target is therefore membership in the center of the supported conserved-sector algebra, together with equality of the terminal fractions across every sector profile sampled by the Bell branches.

---

22. The regulated Bell source-setting-outcome model

Use a finite regulator with source label λ, settings $x, y \in \{0, 1\}$, and outcomes $A, B \in \{-1, +1\}$. For a singlet or Werner source with visibility v_λ, the imported quantum embedding gives

$$q_\lambda(A, B \mid x, y) = \frac{1}{4}[1 - v_\lambda AB\, \mathbf{a}_x \cdot \mathbf{b}_y].$$

With the usual optimal analyzer directions, the singlet has unbiased local marginals and

$$S_0 = 2\sqrt{2}.$$

This is a compatibility construction for the supplied Hilbert space, state, and local instruments. The STF closure number $4\pi^2$ does not derive the singlet, Born rule, or analyzer algebra.

Let a diagonal terminal record effect assign

$$0 \le t_{\lambda xyAB} \le 1$$

to each completed record. Then

$$p_f(\lambda, A, B \mid do(x), do(y)) = \frac{\pi_\lambda q_\lambda(A, B \mid x, y) t_{\lambda xyAB}}{Z_{xy}},$$

where

$$Z_{xy} = \sum_{\lambda, A, B} \pi_\lambda q_\lambda(A, B \mid x, y) t_{\lambda xyAB}.$$

The record calculation separates five conditions.

1. **Full source-and-Born preservation** holds exactly when

   $$t_{\lambda xyAB} = c_{xy} > 0$$

   on every supported branch.

2. **Born preservation at fixed source** requires the weights to be outcome-independent:

   $$t_{\lambda xyAB} = d_{\lambda xy}.$$

3. **Prepared-source preservation** requires only

   $$W_{\lambda xy} := \sum_{A, B} q_\lambda(A, B \mid x, y) t_{\lambda xyAB}$$

   to be independent of λ.

4. **Observational setting-frequency preservation** additionally requires Z_{xy} to be independent of (x, y).

5. **No-signaling** imposes separate normalized marginal equations.

These conditions are logically independent. Source neutrality is weaker than full record neutrality.

---

23. The source-compensated PR obstruction

Define the desired Popescu–Rohrlich sign [22]

$$s_{xy} = -(-1)^{xy}$$

and retain only records satisfying

$$AB = s_{xy}.$$

Rescale the retained weights separately for each source label so every source and setting pair has the same total terminal weight. The resulting diagonal operator is a valid effect with $0 \le t \le 1$.

Theorem 15 — Source-Compensated PR Postselection

There exists a terminal effect that simultaneously:

- preserves the prepared source distribution;
- preserves uniform setting frequencies;
- preserves both no-signaling marginals;
- leaves each local outcome unbiased; and
- produces

  $$E_{00} = E_{01} = E_{10} = -1, \qquad E_{11} = +1,$$

  so that

  $$\boxed{S_f = 4.}$$

This does not violate the quantum Tsirelson theorem. The selected ensemble is not the unconditional output of one normalized state and complete local instruments. It is a globally postselected subensemble of completed records.

The theorem proves that

$$\boxed{\text{measurement independence} + \text{no-signaling} \not\Longrightarrow \text{Born or Tsirelson preservation.}}$$

STF must therefore establish outcome-record neutrality, not merely source neutrality and no-signaling.

If the terminal weights in a setting block lie in $[a_{xy}, b_{xy}]$, define

$$\epsilon_{xy} = \frac{\sqrt{b_{xy}} - \sqrt{a_{xy}}}{\sqrt{b_{xy}} + \sqrt{a_{xy}}}.$$

Then the full source-outcome total-variation error is at most ϵ_{xy}, every marginal shift is no larger, and

$$\boxed{|S_f - S_0| \le 2\sum_{x,y} \epsilon_{xy}.}$$

24. Terminal measurement independence

For each fixed setting intervention, let the effective terminal source weight be $w_{\lambda xy}$. Terminal conditioning gives

$$P_f(\lambda \mid do(x), do(y)) = \frac{P_0(\lambda \mid do(x), do(y))w_{\lambda xy}}{\sum_{\lambda'} P_0(\lambda' \mid do(x), do(y))w_{\lambda' xy}}.$$

Theorem 16 — Terminal Source-Neutrality Criterion

For all prepared source priors, terminal conditioning preserves

$$P_f(\Lambda_S \mid do(x), do(y)) = P_0(\Lambda_S \mid do(x), do(y))$$

if and only if

$$\boxed{w_{\lambda xy} = c_{xy}}$$

on the supported source labels.

The scalar may depend on the externally fixed setting pair because it cancels inside that intervention. If setting frequencies are themselves treated as observational records, equality across (x, y) is an additional requirement.

Full preservation of the Bell experiment is the stronger block condition

$$\boxed{\Pi_{xy} E_R \Pi_{xy} = c_{xy} \Pi_{xy}.}$$

This includes source neutrality and eliminates outcome postselection inside the setting block.

25. The fixed/peripheral algebra controls late-time admissibility

Let Φ_t be the reduced late-time channel on the regulated record system, and suppose the asymptotic dual $\Phi_\infty^\dagger$ exists after peripheral components are treated explicitly. Let $\mathcal{A}_{XY}$ denote the algebra generated only by the setting projectors and the identity on each source-outcome block.

Theorem 17 — Terminal-Effect-Specific Admissibility

A specified terminal macroeffect is Bell-admissible exactly when

$$\boxed{\Phi_\infty^\dagger(P_{\mathrm{HD}}) \in \mathcal{A}_{XY}.}$$

Theorem 18 — Channel-Universal Admissibility

A late-time channel guarantees Bell admissibility for every terminal effect only if

$$\boxed{\mathrm{Ran}(\Phi_\infty^\dagger) \subseteq \mathcal{A}_{XY}.}$$

The first theorem is the physically minimal target: STF has one specified heat-death effect, not every imaginable terminal operator. The second characterizes a stronger channel property.

Pure dephasing fails because it preserves the diagonal source and outcome projectors. Outcome relaxation with the source record fixed also fails source neutrality. In the 32-record regulator, the tested population graphs have fixed-algebra dimensions

$$32, \quad 8, \quad 4, \quad 1,$$

for pure dephasing, outcome-only mixing, one irreducible source-outcome component per setting block, and global primitive relaxation respectively. The third case is the minimal setting-preserving population architecture that guarantees block scalarization.

Noise strength alone does not determine this fixed algebra. The transition graph, exact sectors, and environmental state determine which record observables survive.

---

26. Finite-record decoupling as a conditional route

One proposed route to approximate terminal neutrality uses high-dimensional mixing. Let P_{HD} be a rank-k terminal projector in a d-dimensional accessible shell and let

$$w_r(U) = \mathrm{Tr}\big(P_{\mathrm{HD}} U \sigma_r U^\dagger\big).$$

If the common propagator has Haar second moments, or those of an exact unitary 2-design, then

$$\mathbb{E}\, w_r = f := \frac{k}{d}$$

and

$$\mathrm{Var}(w_r) = \frac{(k - k^2/d)(\mathrm{Tr}\, \sigma_r^2 - 1/d)}{d^2 - 1} \le \frac{f(1-f)}{d+1}.$$

For R branches and ordinary weights q_r,

$$\Pr[\|p_U - q\|_{\mathrm{TV}} > \epsilon] \le \min\left\{1, \frac{R(1-f)(1+\epsilon)^2}{(d+1) f \epsilon^2}\right\}.$$

For fixed R, nonvanishing f, and large d, terminal bias therefore converges to zero in probability under the stated ensemble. This proves a mathematical sufficiency theorem. It does not show that STF produces the ensemble, that the actual one history is typical, or that the exact sector profiles match.

---

27. Why the universal clock is not a unitary design

For one fixed autonomous Hamiltonian,

$$U(t) = e^{-iHt},$$

every propagator commutes with every operator in the commutant of H. Let X be a nonzero traceless conserved operator. Then

$$\mathbb{E}_t[U(t) X U(t)^\dagger] = X,$$

whereas Haar twirling gives zero.

Theorem 19 — Autonomous Temporal-Design No-Go

For $d > 1$, no ensemble generated solely by sampling times under one fixed Hamiltonian is a unitary 1-design, and therefore it cannot be a unitary 2-design.

The result is independent of whether the Hamiltonian is integrable or chaotic. Chaos may improve equilibration of selected observables; it does not remove exact conserved operators from the temporal twirl.

The theorem blocks only the global-design derivation route. The actual STF requirement concerns one terminal observable and one finite branch family. Long-time dephasing gives

$$\overline{w}_r = \mathrm{Tr}[\mathcal{D}_H(P_{\mathrm{HD}})\sigma_r], \qquad \mathcal{D}_H(P) = \sum_E \Pi_E P \Pi_E.$$

A weaker deterministic route therefore remains:

$$\Pi_Q \mathcal{D}_H(P_{\mathrm{HD}})\Pi_Q \simeq f_Q \Pi_Q$$

on the supported sector profiles. Alternatively, a primitive reduced channel may scalarize the terminal effect on the ancient record algebra while global information survives in complementary correlations.

The universal clock orders this evolution. It is not itself the Hamiltonian, random circuit, bath, or mixing ensemble.

---

28. FP V8.1 response sector

The canonical clock-relative curvature variable is

$$\boxed{q_N = \mathcal{R}_{\mathrm{STF}}(N) = \sqrt{R^2 + 8\mathcal{W}_N}.}$$

The exact selective response is

$$\boxed{K^R_{\mathrm{sel}}(t) = \delta(t) - \omega_c e^{-\omega_c t}\Theta(t),}$$

or, in frequency space,

$$\boxed{K_{\mathrm{sel}}(\omega) = -\frac{i\omega}{\omega_c - i\omega}.}$$

It has zero static response:

$$K_{\mathrm{sel}}(0) = 0.$$

The subtracted Ohmic–Drude response

$$D_R^{\mathrm{sub}}(\omega) = -i\eta\omega \frac{\Lambda_B}{\Lambda_B - i\omega}$$

matches the V8.1 kernel exactly under

$$\Lambda_B = \omega_c, \qquad \eta = \omega_c^{-1}.$$

This supplies an exact Gaussian-bath realization of the response shape. It does not derive the microscopic compactification bath or its quantum state.

The dimensionless coupling

$$\kappa = \frac{\zeta/\Lambda}{L_*^2} \simeq 10^{70}$$

is not a record-mixing rate. A reduced relaxation rate also depends on the environmental state, spectral density, system operator, exact sectors, and coarse graining.

Because V8.1 has $\omega_c \sim m_s$, the 3.32-year mode lies in the full-memory regime $\omega/\omega_c = O(1)$. A GKSL semigroup is therefore a finite comparison regulator, not the physical completion. The correct object is a non-Markovian process tensor or exact finite-time channel built from the full kernel.

---

29. Response does not determine noise or record transfer

A closed compactification with deterministic degrees of freedom does not automatically produce a stochastic noise kernel after reduction. To derive an influence functional, STF must specify environmental fields, their state, and their couplings to the system variables.

For a Gaussian environment, the retarded response and symmetric noise are distinct state-dependent kernels. Positivity constrains their spectral matrix. A cross-only noise ansatz is generally indefinite; reciprocal diagonal terms are needed for a positive semidefinite shared-bath completion.

The exact response coefficient does not identify:

- the environmental state;
- the symmetric noise kernel N_K;

- reciprocal metric-clock backreaction;
- the visible production operator;
- the record population-transition operator; or
- the late-time spectral gap.

Once a population generator is supplied, the connected components of its transition graph determine the diagonal fixed algebra. In the regulated Bell model, irreducibility inside each setting block is the minimal graph property that removes source and outcome projectors while preserving the intervention labels.

This establishes a crucial distinction:

$$\boxed{\text{response selectivity} \neq \text{noise} \neq \text{irreversible record transfer.}}$$

---

30. The zero-mode memory obstruction

The V8.1 selective kernel admits the one-auxiliary localization

$$(K_{\mathrm{sel}} * x)(t) = x(t) - z(t), \qquad \dot{z} = \omega_c(x - z).$$

Suppose, only as a test, that this same high-pass kernel directly drives a record mode with graph-Laplacian eigenvalue $\lambda > 0$ and coupling Γ. The asymptotic mode retention is

$$\boxed{r_\lambda = \frac{\omega_c}{\omega_c + \Gamma\lambda} > 0}$$

at every finite coupling.

Theorem 20 — Zero-Mode Memory Obstruction

Direct identification of the zero-static selective response with the sole record-population relaxation kernel produces a full-rank asymptotic resolvent and cannot yield exact channel-universal scalarization at finite coupling.

For a CHSH tolerance of 10^{-3}, the regulated calculation requires

$$\frac{\Gamma \lambda_{\mathrm{gap}}}{\omega_c} \geq 2696.26.$$

This is an approximate suppression condition, not an exact erasure theorem. The value $\kappa \sim 10^{70}$ cannot be substituted for the left-hand ratio.

Theorem 21 — Two-Kernel Obligation

A completed STF open-system sector must distinguish at least:

$$\boxed{K_{\mathrm{sel}}^{R} \quad \text{for static-curvature selectivity}}$$

and

$$\boxed{M_{\mathrm{mix}} \quad \text{for irreversible source-outcome record transfer.}}$$

The two kernels may arise from one reciprocal microscopic environment, but their equality is not implied and is inconsistent with exact finite-coupling record erasure under the direct-identification model.

This is not a no-go for STF. It shows that the high-pass property needed to avoid static curvature response and the low-frequency dissipation needed to erase accessible record populations are different functional obligations.

---

31. The unavoidable production map

The regulated Bell algebra is a target until STF supplies the physical map that creates visible records. The theory requires a microscopic operator or instrument

$$O_{\mathrm{prod}}$$

that maps the STF scalar, clock-relative curvature, matter sector, detector, and environment into source and outcome records.

The correct completion chain is

$$\boxed{\begin{array}{l}\text{10D parent} \longrightarrow (q_N, \kappa, K^R_{\text{sel}})_{\text{fixed clock}} \\ \longrightarrow \text{microscopic environment and } N_K \\ \longrightarrow O_{\text{prod}} \text{ and } M_{\text{mix}} \\ \longrightarrow \text{non-Markovian process tensor} \\ \longrightarrow \text{fixed/peripheral algebra} \\ \longrightarrow E_R = \mathcal{J}^\dagger(U^\dagger P_{\text{HD}} U).\end{array}}$$

The same physical derivation must retain environmental memory that remains visible to the terminal effect. Tracing out a branch-correlated environment before evaluating P_{HD} would falsely manufacture scalarization.

Part III — The Combined Two-Boundary Theorem

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32. The Two-Boundary Bell-Autonomy Theorem

The initial and terminal problems now have parallel operator forms:

$$K_{xy} = q_{xy} I_S$$

prevents the setting process from resolving the source state, while

$$\Pi_{xy} E_R \Pi_{xy} = c_{xy} \Pi_{xy}$$

prevents the future macrocondition from resolving source or outcome records inside the completed setting block.

Theorem 22 — Operationally Non-Superdeterministic STF Bell Completion

Let Λ_S be the gauge-invariant source-world-tube state, defined independently of later settings, outcomes, and the complete history. Suppose:

1. **Well-defined initial setting process.** The complete setting sector induces effects K_{xy} on the supported source algebra.
2. **Initial setting-effect scalarization.** For every supported pair,

$$K_{xy} = q_{xy} I_S, \qquad q_{xy} > 0.$$

3. **Wing structure.** When independent settings are required,

$$q_{xy} = q_X(x)q_Y(y).$$

4. **Physical modularity.** Natural settings and forced settings are charge-preserving local replacements of the same device modules.
5. **Preterminal source invariance.** Replacing those modules leaves the source process unchanged:

$$\rho_S[\mathcal{M}_X, \mathcal{M}_Y] = \rho_S.$$

6. **Complete local instruments.** The supplied local quantum instruments are normalized and operationally no-signaling before terminal conditioning.
7. **Terminal setting-block scalarization.** The physical pulled-back heat-death effect satisfies

$$\Pi_{xy} E_R \Pi_{xy} = c_{xy} \Pi_{xy}, \qquad c_{xy} > 0,$$

on every supported source-outcome block.

Then

$$\boxed{I_0(\Lambda_S : X, Y) = 0,}$$

$$\boxed{P_f(\Lambda_S \mid do(x), do(y)) = P_0(\Lambda_S),}$$

and

$$\boxed{P_f(\Lambda_S, A, B \mid do(x), do(y)) = P_0(\Lambda_S) q(A, B \mid \Lambda_S, x, y).}$$

The completed STF Bell experiment is therefore non-superdeterministic in the operational Bell sense defined by its physical source and setting interventions.

Proof

Premise 2 makes the natural setting probabilities independent of every supported source state. Premises 3 and the support condition give the required wing sampling. Premises 4 and 5 give physical intervention autonomy, so the preterminal source distribution is unchanged when either setting module is replaced. Premise 7 assigns one scalar terminal weight to every supported source-outcome state inside a fixed setting intervention. That scalar cancels from the terminal normalization. The supplied source prior and conditional quantum outcome distribution therefore remain unchanged. □

The theorem is sufficient and, within the universal all-state architecture, its scalarization conditions are also necessary. For one restricted realized ensemble, weaker history-balance or finite-family neutrality equalities can hold accidentally. Such ensemble-specific cancellation is not a universal autonomy proof.

33. Approximate two-boundary autonomy

Exact scalarization is an ideal theorem target. Experiments test finite tolerances.

Let the initial setting process obey

$$\varepsilon_{\mathrm{set}} = \sup_{\rho_S} \frac{1}{2} \left\| \mathcal{C}_{\mathrm{set}}(\rho_S) - \mathcal{R}_q(\rho_S) \right\|_1,$$

with

$$\varepsilon_{\mathrm{set}} \leq \delta_{\mathrm{micro}} + \delta_{\mathrm{center}}$$

or, in an explicit reset dilation,

$$\varepsilon_{\mathrm{set}} \leq \delta_{\mathrm{anc}} + \eta_X + \eta_Y.$$

These are alternative resolutions of the same black-box error and should not be added if they double-count one imperfection.

Let the terminal effect in setting block (x, y) have spectral error

$$\epsilon_{xy} = \frac{\sqrt{b_{xy}} - \sqrt{a_{xy}}}{\sqrt{b_{xy}} + \sqrt{a_{xy}}}.$$

Then the completed source-outcome distribution differs from the ideally autonomous experiment by at most the initial preparation error plus the terminal block error under the usual total-variation telescoping bound. The terminal contribution to CHSH obeys

$$|S_f - S_0| \leq 2 \sum_{x,y} \epsilon_{xy}.$$

A complete experimental claim must state separately:

- initial source-to-setting leakage;
- intervention-induced source shift;
- wing correlation and support errors;
- terminal source-outcome distortion;

- induced signaling; and
- Bell-functional distortion.

No single entropy, mutual-information, or decoherence number substitutes for this vector of diagnostics.

---

34. What the two-clock architecture proves—and what it does not

Clock separation is essential to the theorem's formulation.

1. It permits a source state on an intermediate universal-time leaf to be defined without importing the later settings.
2. It permits local setting and outcome records to remain meaningful on internal clocks while a universal boundary evaluates the completed history.
3. It distinguishes universal ordering from local operational duration.
4. It prevents the oscillating internal scalar phase from being misused as a globally monotonic clock.

But the two clocks do not prove:

- that the setting resources are fresh;
- that the setting effects are scalar;
- that the terminal effect is record neutral;
- that the universal clock randomizes the global state;
- that a mixing depth $\gamma\Delta T_U$ corresponds to a specified number of internal-clock years; or
- that one actual history is typical in a hypothetical ensemble.

The two clocks make the autonomy problem well posed. The source, setting, and terminal dynamics must still solve it.

---

35. Does STF prove superdeterminism?

No.

No displayed FP V8.1 term forces

$$I_0(\Lambda_S : X, Y) > 0,$$

forces failure of local setting modularity, or makes the physical terminal effect non-scalar on Bell records. The universal clock supplies ordering, not a source-setting conspiracy. Global closure supplies a possible terminal weighting mechanism, not a theorem that the weighting distinguishes the records. One actual history contains the actual settings, but that tautology does not identify a Bell source variable.

The framework is compatible with an imported singlet, independent local instruments, unbiased marginals, and $S = 2\sqrt{2}$. That construction proves consistency, not physical derivation.

Therefore

$$\boxed{\mathrm{STF}_{\mathrm{V8.1}} \nvdash \mathrm{SD}.}$$

---

36. Does STF disprove superdeterminism?

Not yet.

The initial mathematical architecture is exact, but FP V8.1 has not produced the setting effects K_{xy} or proved source invariance under the forced modules. The terminal mathematical architecture is exact, but FP V8.1 has not produced the physical E_R or proved setting-block scalarization.

Both of the following remain compatible with the incomplete action:

1. a completion satisfying Theorem 22;
2. a completion in which a common-past sector leaks into the settings, a forced module changes the source, or the terminal effect retains a source/outcome projector.

Therefore

$$\boxed{\mathrm{STF}_{\mathrm{V8.1}} \nvdash \neg\mathrm{SD}.}$$

This is not an unresolved definition. It is an unresolved physical calculation with exact pass/fail criteria.

---

37. FP V8.1 status ledger

OBJECT OR CLAIM	STATUS AFTER THE SYNTHESIS
Bell-relevant source world tube	operationally defined
Complete actual history as Λ_S	rejected as tautological
History-balance criterion	theorem
Fresh stochasticity alone implies independence	false; explicit counterexample
Replacer is unique universal decoupler	theorem
Sector-preserving neutral-pointer module	explicit construction
Exact finite-time bounded Markovian reset	impossible
Setting-effect scalarization $K_{xy} = q_{xy} I_S$	theorem target
K_{xy} derived from FP V8.1	open
Physical modular setting interventions derived	open
Complete terminal branch-neutrality criterion	theorem
Pulled-back effect E_R controls all terminal weights	theorem
Setting-block scalarization preserves the Bell experiment	theorem
Source independence plus no-signaling implies quantum correlations	false; PR counterexample
Haar/2-design concentration route	conditional theorem
One autonomous Hamiltonian supplies a unitary design	false
Exact V8.1 response-kernel form	specified
Gaussian-bath realization of that response shape	constructed

OBJECT OR CLAIM	STATUS AFTER THE SYNTHESIS
Microscopic environment and state	open
State-dependent noise kernel N_K	open
Visible production operator O_{prod}	open
Irreversible record-transfer kernel M_{mix}	open
$K_{\mathrm{sel}}^R = M_{\mathrm{mix}}$	not derived; direct identification obstructed
Non-Markovian record process tensor	open
Physical fixed/peripheral algebra	open
Physical pulled-back heat-death effect E_R	open
Dynamical universal clock carrier	open in FP V8.1
STF V8.1 proves superdeterminism	no
STF V8.1 proves non-superdeterminism	no

38. Falsifiers and decisive success conditions

The non-superdeterministic completion fails if the derived physical theory produces any of the following without an exact compensating theorem:

1. non-scalar source-to-setting effects,

$$K_{xy} \not\propto I_S;$$

2. within-sector or center leakage from the source into the setting labels;
3. a natural or forced setting module that changes the source preparation;
4. missing setting-pair support or unaccounted wing correlation;
5. a terminal source effect that depends on λ inside a fixed intervention;
6. a terminal effect retaining source or outcome projectors in a setting block;
7. unequal terminal fractions across branch-dependent exact-sector profiles;
8. a derivation that traces out environmental record information still visible to P_{HD};

9. a record dynamics consisting only of dephasing;
10. direct use of K^R_{sel} as exact finite-coupling population erasure;
11. controllable advanced-sector signaling; or
12. terminally conditioned correlations outside the quantum set.

The completion succeeds if STF derives:

$$\boxed{K_{xy} = q_{xy} I_S}$$

for the physical natural setting process, verifies source invariance under the charge-preserving forced modules, and derives

$$\boxed{\Pi_{xy} E_R \Pi_{xy} = c_{xy} \Pi_{xy}}$$

for the actual regulated heat-death effect on every supported Bell block.

---

39. Decisive completion program

The next calculations are ordered by dependency.

39.1 Initial setting sector

1. Identify the physical setting-generator degrees of freedom and their world tubes.
2. Construct the gauge-invariant upstream source and setting algebras, including their shared center.
3. Derive the setting resources, their sector-conditioned state, and their renewal mechanism.
4. Derive the natural and forced pointer modules.
5. Pull the setting readout backward to calculate K_{xy}.
6. Test separately

$$K_{xy} - \mathbb{E}_3(K_{xy}) = 0$$

and

$$\mathbb{E}_3(K_{xy}) - q_{xy} I = 0.$$

7. Verify preterminal source invariance under each forced module.

39.2 Terminal record sector

1. Derive the visible production operator O_{prod}.
2. Specify the microscopic environment and its state.
3. Derive the response/noise spectral matrix, including reciprocal metric-clock response.
4. Derive M_{mix} rather than identifying it with K^R_{sel}.
5. Construct the non-Markovian reduced process tensor.
6. Calculate its exact conserved sectors and fixed/peripheral algebra.
7. Define the physical regulated terminal macroeffect P_{HD}.
8. Calculate

$$E_R = \mathcal{J}^\dagger(U^\dagger P_{\mathrm{HD}} U).$$

9. Diagonalize or block-reduce E_R on every supported setting sector.
10. Compare the resulting scalarization error with Bell and no-signaling tolerances.

Only after both branches close may the conditional theorem be upgraded to a proof about the physical STF completion.

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40. The thirteen-paper dependency map

This paper is Version 2.0 of *Superdeterminism in the Two-Clock STF Framework* (V1.0, 2026), which it supersedes as the public statement of the program. Counting that superseded status paper, the sequence comprises thirteen papers; the synthesis absorbs the conclusions, not the full technical bulk, of the twelve companions listed below together with the alignment note.

Superseded status paper

0. *Superdeterminism in the Two-Clock STF Framework*, V1.0 — replaced by the present synthesis.

Initial-autonomy branch

1. *The STF Source–Setting Autonomy Theorem*.
2. *The STF Setting-Generator Decoupling Theorem*.
3. *The STF Fresh-Ancilla and Reset-Resource Audit*.

4. *The STF Setting-Sector and Boundary-Center Construction*.

Terminal-neutrality branch

5. *STF Terminal Neutrality and Measurement Independence Audit*.
6. *The STF Record-Algebra Terminal-Scalarization Theorem*.
7. *The STF Regulated Bell Source–Setting–Record Audit*.
8. *The STF Late-Time Fixed-Point Algebra Theorem*.
9. *The STF Influence-Functional and Noise-Kernel Closure Audit*.
10. *The STF Zero-Mode Memory Obstruction and Two-Kernel Theorem*.
11. *STF Late-Time Sectorwise Mixing Feasibility Audit*.
12. *The Finite-Record Terminal-Decoupling Theorem*.

Alignment document

13. *STF FP V8.1 Alignment Note*.

The alignment note is incorporated into Sections 28–31 and 37. It need not remain a separate public research claim, although it should remain in the version archive.

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41. Reproducibility map

The following companion calculations test the finite-dimensional statements used here:

1. `stf_source_setting_autonomy_checks.py`
2. `stf_setting_generator_decoupling_checks.py`
3. `stf_fresh_ancilla_reset_resource_checks.py`
4. `stf_setting_sector_boundary_center_checks.py`
5. `stf_terminal_neutrality_and_measurement_independence_tests.py`
6. `stf_record_algebra_scalarization_checks.py`
7. `stf_regulated_bell_terminal_effect_checks.py`
8. `stf_fixed_point_bell_channel_checks.py`

9. `stf_influence_kernel_record_connectivity_checks_fp8_1.py`
10. `stf_zero_mode_memory_obstruction_checks.py`
11. `stf_late_time_mixing_feasibility_checks.py`
12. `stf_finite_record_terminal_decoupling_checks.py`
13. `stf_superdeterminism_status_paper_checks.py`

Together they reproduce:

- the 0.2780719051-bit biased-fresh-innovation counterexample;
- the 0.6813316047-bit common-cause counterexample with zero intervention shift;
- the $1/2$ fine-tuned retrocausal intervention shift;
- replacer-channel decoupling and the SWAP correlation transfer;
- the exact fresh-resource error identity;
- the bounded finite-time reset no-go examples;
- the microstate/center leakage decomposition;
- the record-commutant counterexample;
- the sharp terminal probability-distortion bound;
- the source-compensated PR effect with $S = 4$;
- the fixed-algebra dimensions $32, 8, 4, 1$;
- the exact Drude/V8.1 response-kernel match;
- the zero-mode residual and the 2696.26 suppression target;
- the autonomous temporal-design obstruction; and
- the finite-record concentration and conserved-sector tests.

These calculations verify the regulated theorems and counterexamples. They do not simulate the missing cosmological setting sector or derive the physical terminal macroeffect.

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42. References

STF framework and companion papers

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4. Z. Paz, *One Bit of Destiny — The Theorem of the Unknown*, V1.0 (2026), https://existshappens.com/papers/one-bit-of-destiny/.
5. Z. Paz, *The STF Source–Setting Autonomy Theorem*, V1.0 (2026).
6. Z. Paz, *The STF Setting-Generator Decoupling Theorem*, V1.0 (2026).
7. Z. Paz, *The STF Fresh-Ancilla and Reset-Resource Audit*, V1.0 (2026).
8. Z. Paz, *The STF Setting-Sector and Boundary-Center Construction*, V1.0 (2026).
9. Z. Paz, *STF Terminal Neutrality and Measurement Independence Audit*, V1.0 (2026).
10. Z. Paz, *The STF Record-Algebra Terminal-Scalarization Theorem*, V1.0 (2026).
11. Z. Paz, *The STF Regulated Bell Source–Setting–Record Audit*, V1.1 (2026).
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Bell theory, causality, and cosmic setting tests

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43. Final theorem boundary

The complete result can be stated in two lines:

$$\boxed{K_{xy} = q_{xy} I_S \quad + \quad \text{physical source-invariant module replacement}}$$

closes the initial source-setting branch, and

$$\boxed{\Pi_{xy} \mathcal{J}^{\dagger}(U^{\dagger} P_{\mathrm{HD}} U) \Pi_{xy} = c_{xy} \Pi_{xy}}$$

closes the terminal source-outcome branch.

If both are derived from the physical STF theory, the completed Bell experiment is operationally non-superdeterministic. If the first fails, STF contains initial source-setting dependence or nonmodular settings. If the second fails, the terminal boundary reweights Bell records. If neither object is calculated, the ontology is undecided regardless of how deterministic, global, or time-symmetric its language appears.

The present framework has proved the equivalences, the sufficient physical architectures, the finite error bounds, and the principal no-go results. It has not yet derived the two decisive effects from FP V8.1.

Therefore the exact answer is

$$\boxed{\begin{array}{l} \mathrm{STF}_{\mathrm{V8.1}} \nvdash \mathrm{SD}, \\ \mathrm{STF}_{\mathrm{V8.1}} \nvdash \neg\mathrm{SD}, \\ \text{a non-superdeterministic STF completion is mathematicallyviable,} \\ \text{and its remaining proof obligations are now explicit.} \end{array}}$$

That is the substantive outcome of the thirteen-paper sequence. The question has been reduced from metaphysics to two operator calculations.

CITATION

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